PIE Tech
POLLACHI INSTITUTE OF ENGINEERING AND TECHNOLOGY
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Degree/Branch: B.E / ECE **Semester / Year : V/III**

Subject Code / Name: EC8553/ Discrete Time Signal Processing

(Question Bank)

UNIT - I

DISCRETE FOURIER TRANSFORM

PART A (2 Marks)

1. What is zero padding ? What are it uses. Nov/Dec 13

* Let the sequence x(n) has a length L.
* If we want to find N point DFT (N>L) of sequence x(n), we have to add (N-L) zeros to the sequence x(n)
* This is known as zero padding.

⊛ Zero padding is used in
1) Calculation of DFT
2) Linear Filtering
3) Better display of the frequency spectrum.

2. How many multiplications and additions are required to compute N-point DFT using DFT and radix 2 FFT. Nov/Dec 13 Dec 12 Dec 13

* For	DFT	FFT
No. of additions	$N(N-1)$	$N \log_2 N$
No. of Multiplications.	N^2.	$\frac{N}{2} \log_2 N$.

3. State the difference between DFT and DTFT.

Dec 12

DTFT (Discrete Time Fourier Transform)	DFT.
* Time domain sequence is discrete but frequency domain representation is continuous	Both time domain sequence and frequency domain representation are discrete.
DTFT cannot be evaluated using fast algorithms.	DFT can be evaluated using fast algorithms.
* DTFT is continuous version of DFT.	* DFT is discrete version of DTFT.

4. What is Bit reversal?

* In case of DFT FFT algorithm, the input sequence $x(n)$ is applied in bit reversal order.

* For example each n is represented by 3 bits n_2, n_1, n_0 then the bit reversal value of n will be n_0, n_1, n_2.

eg:

	Bit reversal
$x(0) = 000$	$x(0) = 000$.
$x(1) = 001$	$100 = x(4)$
$x(2) = 010$	$010 = x(2)$
$x(3) = 011$	$110 \quad x(6)$

5. Is the DFT of a finite length sequence periodic? If so, state the theorem? APR/May 2015

If $x(n)$ is periodic and $X(k)$ is also periodic.

If $x(n) = x(n+N)$ for all n.

$X(k) = X(k+N)$ for all k.

Proof: $X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn}$

Put $k = k+N$.

$$X(k+N) = \sum_{n=0}^{N-1} x(n) W_N^{(k+N)n}$$

$$= \sum_{n=0}^{N-1} x(n) W_N^{kn} W_N^{nN}$$

$$= \sum_{n=0}^{N-1} x(n) \cdot W_N^{kn}.$$

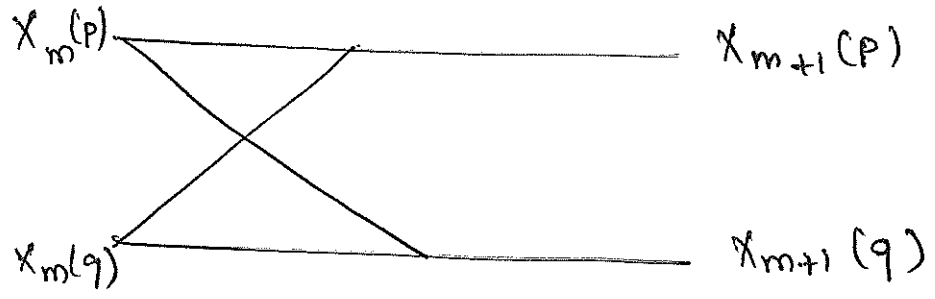
$W_N = e^{-j2\pi/N}$

$W_N^{nN} = e^{\frac{-j2\pi nN}{N}}$

$= 1.$

$X(k+N) = X(k)$. Hence proved

6. What is meant by 'in place computations' in DIT and DIF algorithms? June 12



* The nodes $X_m(p)$ and $X_m(q)$ represents memory locations of inputs where the input values are stored.

* After the outputs $X_{m+1}(p)$ and $X_{m+1}(q)$ are calculated, the same memory location is used to store the new values in place of the input values.

* In DIF & DIT algorithm use the same location to store the input and output sequence is called "in place" algorithm.

7. Define DFT and IDFT (or) Define DFT pair (or) Write the analysis and synthesis equations of DFT. APR/May 2015

DFT (analysis) : $X(k) = \sum_{n=0}^{N-1} x(n) e^{\frac{-j2\pi nk}{N}}$ $k = 0, 1, .. N-1$

IDFT (synthesis) : $x(n) = \frac{1}{N} \sum_{k=0}^{N-1} x(k) e^{\frac{j2\pi nk}{N}}$ $n = 0, 1, .. N-1$

8. What is twiddle factor? Dec 12

The twiddle factor is given by,

$$W_N = e^{-j2\pi/N}.$$

DFT : $X(k) = \sum_{n=0}^{N-1} x(n) . W_n^{kn}$ $0 \le k \le N-1$.

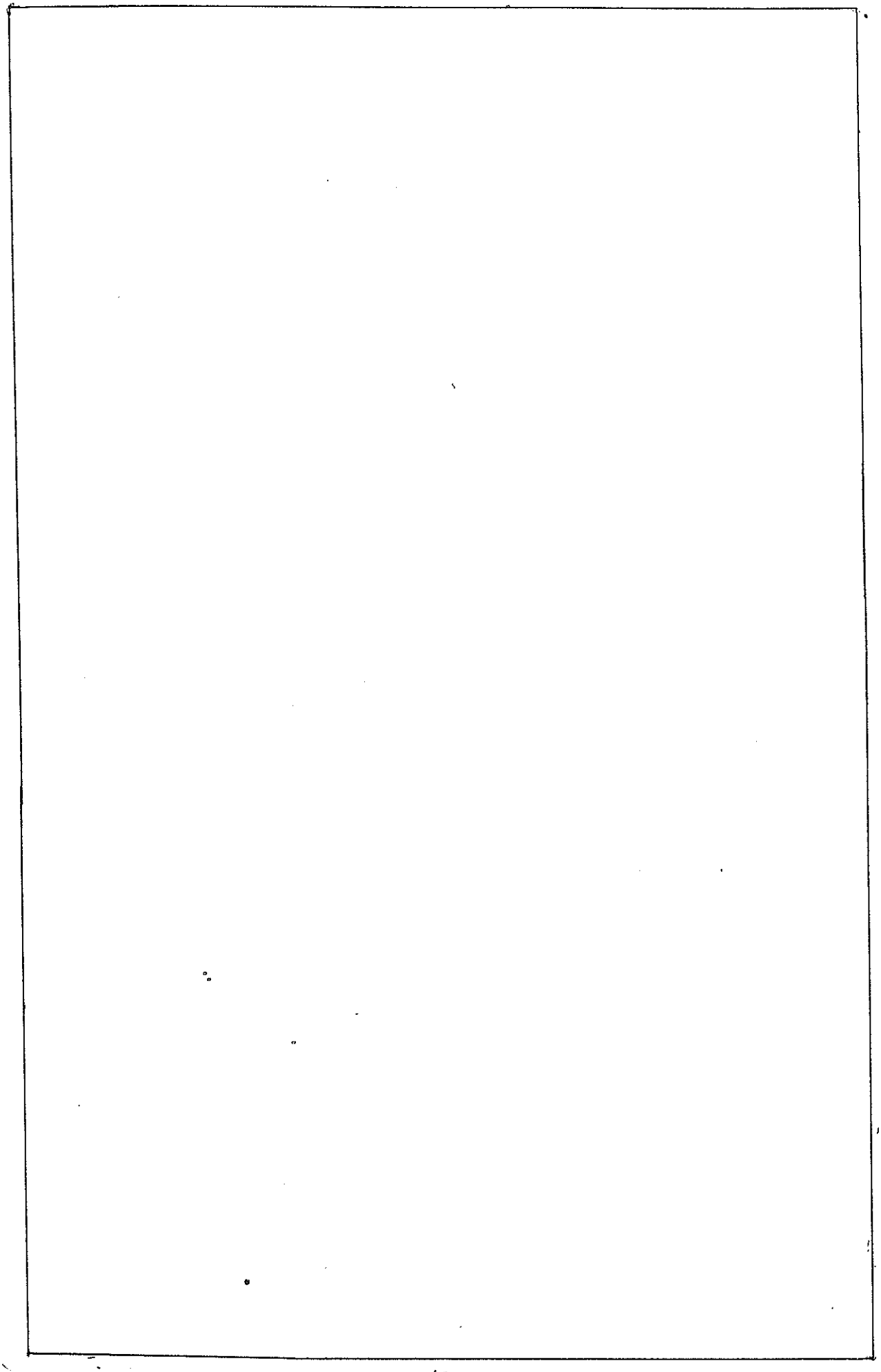
IDFT $x(n) = \frac{1}{N} \sum_{n=0}^{N-1} x(k) W_N^{-kn}$ $0 \le n \le N-1$

9. What is meant by radix-2 FFT algorithm

* In radix 2 FFT algorithm, the decimation arrives finally at 2 point sequence.
* The 2 point DFT is calculated by direct computation.

10. What are the differences between DIF and DIT algorithms? APR/MAY 2015-

DIT (Decimation in Time)	DIF. (Decimation in frequency)
* Input is bit reversal while output in natural order.	* Input is natural order while the output is bit reversal.
* Complex multiplications takes place before add and subtract operations.	Complex multiplications takes place after add and subtract operation.
* The N point DFT is split in two $\frac{N}{2}$ point DFT.	The N point DFT is split in two $\frac{N}{2}$ point DFT.



UNIT - I.

Part - B.

1. Determine the 8 point DFT of the sequence.

$x(n) = \{1, 1, 1, 1, 1, 1, 0, 0\}$.

APR/MAY 2015
Dec 13.
May/June 13
Dec 12

Soln:

DFT of $x(n)$ is given by.

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{\frac{-j2\pi nk}{N}} \quad ; k = 0, 1, \ldots N-1.$$

$N = 8$

Here, $x(0) = x(1) = x(2) = x(3) = x(4) = x(5) = 1$

$x(6) = x(7) = 0$.

$$X(k) = \sum_{n=0}^{7} x(n)\, e^{\frac{-j2\pi nk}{8}} = \sum_{n=0}^{7} x(n)\, e^{\frac{-j\pi nk}{4}}.$$

$$= x(0) + x(1)\, e^{\frac{-j\pi k}{4}} + x(2)\, e^{-j\pi k/2} + x(3)\, e^{-j3\pi k/4}$$
$$+ x(4)\, e^{-j\pi k} + x(5)\, e^{-j5\pi k/4} + x(6)\, e^{-j6\pi k/4}$$
$$+ x(7)\, e^{-j7\pi k/4}.$$

$$X(k) = 1 + e^{\frac{-j\pi k}{4}} + e^{-j\pi k/2} + e^{\frac{-j3\pi k}{4}} + e^{-j\pi k} + e^{-j5\pi k/4}.$$

$$= 1 + \left[\cos\frac{\pi k}{4} - j\sin\frac{\pi k}{4}\right] + \left[\cos\frac{\pi k}{2} - j\sin\frac{\pi k}{2}\right]$$
$$+ \left[\cos\frac{3\pi k}{4} - j\sin\frac{3\pi k}{4}\right] + \left[\cos\pi k - j\sin\pi k\right] +$$
$$\left[\cos\frac{5\pi k}{4} - j\sin\frac{5\pi k}{4}\right].$$

Put $K = 0$

$$X(0) = 1 + (1-0) + (1-0) + (1-0) + (1+0) + (1-0)$$

$$\boxed{X(0) = 6.}$$

Put $K = 1$

$$X(1) = 1 + \left(\cos\frac{\pi}{4} + j\sin\frac{\pi}{4}\right) + \left(\cos\pi/2 - j\sin\pi/2\right) + \cos\frac{3\pi}{4} - j\sin\frac{3\pi}{4}$$

$$+ \cos\pi - j\sin\pi) + \cos\frac{5\pi}{4} - j\sin\frac{5\pi}{4}\Big)$$

$$= 1 + 0.707 - j0.707 + 0 - j - 0.707 - j0.707 - 1 - 0.707 + j0.707$$

$$\boxed{X(1) = -0.707 - j1.707}$$

Put $K = 2$

$$X(2) = 1 + (\cos\pi/2 - j\sin\pi/2) + (\cos\pi - j\sin\pi) + (\cos\frac{3\pi}{2} - j\sin 3\pi/2)$$

$$+ (\cos 2\pi - j\sin 2\pi) + (\cos 5\pi/2 - j\sin 5\pi/2)$$

$$= 1 - j - 1 + j + 1 - j$$

$$\boxed{X(2) = 1 - j.}$$

Put $K = 3.$

$$X(3) = 1 + \left(\cos\frac{3\pi}{4} - j\sin\frac{3\pi}{4}\right) + \left(\cos\frac{3\pi}{2} - j\sin\frac{3\pi}{2}\right)$$

$$+ \left(\cos\frac{9\pi}{4} - j\sin\frac{9\pi}{4}\right) + (\cos 3\pi - j\sin 3\pi)$$

$$+ \left(\cos\frac{15\pi}{4} + j\sin 15\pi/4\right)$$

$$\boxed{X(3) = 0.707 + j0.293.}$$

Put $K = 4$.

$$x(4) = 1 + (\cos\pi - j\sin\pi) + (\cos 2\pi - j\sin 2\pi) + (\cos 3\pi - j\sin 3\pi) + (\cos 4\pi - j\sin 4\pi) + (\cos 5\pi - j\sin 5\pi)$$

$$= 1 - 1 + 1 - 1 + 1 - 1$$

$$\boxed{x(4) = 0}$$

Put $K = 5$.

$$x(5) = 1 + (\cos 5\pi/4 - j\sin 5\pi/4) + (\cos 5\pi/2 - j\sin 5\pi/2) + (\cos 15\pi/4 - j\sin 15\pi/4] + (\cos 5\pi - j\sin 5\pi) + (\cos \frac{25\pi}{4} - j\sin 25\pi/4)$$

$$= 1 - 0.707 + j0.707 - j + 0.707 + j0.707 - 1 + 0.707 - j0.707$$

$$\boxed{x(5) = 0.707 - j0.293}$$

Put $K = 6$.

$$x(6) = 1 + (\cos 3\pi/2 - j\sin 3\pi/2) + (\cos 3\pi - j\sin 3\pi) + (\cos 9\pi/2 - j\sin 9\pi/2) + (\cos 6\pi - j\sin 6\pi) + (\cos 15\pi/2 - j\sin 15\pi/2)$$

$$\boxed{x(6) = 1 + j}$$

Put $K = 7$.

$$X(7) = 1 + \left(\cos\frac{7\pi}{4} - j\sin\frac{7\pi}{4}\right) + \left(\cos 7\pi/2 - j\sin 7\pi/2\right)$$
$$+ \left(\cos\frac{21\pi}{4} - j\sin 21\pi/4\right) + \left(\cos 7\pi - j\sin 7\pi\right)$$
$$+ \left(\cos\frac{35\pi}{4} - j\sin\frac{35\pi}{4}\right)$$

$$\boxed{X(7) = -0.707 + j1.707.}$$

Result: The DFT of the sequence

$$x(k) = \{6, -0.707 - j1.707, 1 - j, 0.707 + j0.293, 0,$$
$$0, 0.707 - j0.293, 1 + j, -0.707 + j1.707\}$$

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2. Perform Circular Convolution of the following sequence, $x(n) = \{1, 1, 2, 1\}$ and $h(n) = \{1, 2, 3, 4\}$. Using DFT and IDFT Method. June 13.

Soln:

Let

$$X_3(k) = X_1(k)\, X_2(k)$$

$$\text{IDFT}\,[X_3(k)] = x(n).$$

Given $x_1(n) = \{1, 2, 2, 1\}$.

The formula for DFT is, $X_1(k) = \left[W_N^{kn}\right] x_1(n)$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1+2+1 \\ 1-j-2+j \\ 1-1+2-1 \\ 1+j-2-j \end{bmatrix} = \begin{bmatrix} 5 \\ -1 \\ 1 \\ 1 \end{bmatrix}$$

$$x_1(k) = \{5, -1, 1, 1\}$$

And also $y(n) = \{1, 2, 3, 4\}$.

Let $x_2(n) = y(n) = \{1, 2, 3, 4\}$.

$$x_2(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 1+2+3+4 \\ 1-2j-3+4j \\ 1-2+3-4 \\ 1+2j-3-4j \end{bmatrix}$$

$$x_2(k) = \{10, -2+2j, -2, -2-2j\}$$

Now, $X_3(k) = X_1(k)\, X_2(k)$

$$= \{50, 2-2j, -2, -2+2j\}$$

To find $x_3(n)$

$$x_3(n) = \left[W_N^{Kn} \right] X_3(K).$$

$$= \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 50 \\ 2-2j \\ -2 \\ 2+2j \end{bmatrix}$$

$$\boxed{x_3(n) = \{13, 14, 11, 12\}}$$

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3. Derive the expression for Radix-2 DIT-FFT algorithm.

Proof:

In DIT, time domain sequences $x(n)$ is splitted into several number of sequences. So it is called decimation in Time:

Let the N point data sequence $x(n)$ be splitted into $N/2$ point data sequences $f_1(n)$ and $f_2(n)$.

$f_1(n)$ = Contains even numbered sample of $x(n)$

$f_2(n)$ = Contains odd numbered sample of $x(n)$.

We can write,

$$f_1(n) = x(2n) \quad ; \quad n = 0, 1, \ldots \frac{N}{2} - 1.$$

$$f_2(n) = x(2n+1) \quad ; \quad n = 0, 1, \ldots \frac{N}{2} - 1.$$

N point DFT is given as,

$$X(K) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j2\pi nK/N}. \qquad K = 0, 1, \ldots N-1.$$

$$= \sum_{n=0}^{N-1} x(n)\, W_N^{Kn}$$

$x(n)$ splitted into two sequences

$$X(K) = \sum_{n=\text{even}} x(n)\, W_N^{Kn} + \sum_{n=\text{odd}} x(n)\, W_N^{Kn}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x(2n)\, W_N^{2Kn} + \sum_{n=0}^{\frac{N}{2}-1} x(2n+1)\, W_N^{K(2n+1)}$$

We know that-

$$f_1(n) = x(2n) \quad \text{and} \quad f_2(n) = x(2n+1)$$

$$= \sum_{n=0}^{\frac{N}{2}-1} f_1(n)\, W_N^{2nK} + \sum_{n=0}^{\frac{N}{2}-1} f_2(n) \cdot W_N^{2nK} \cdot W_N^{K}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} f_1(n)\, W_{N/2}^{Kn} + \sum_{n=0}^{\frac{N}{2}-1} f_2(n)\, W_{N/2}^{Kn}\, W_N^{K}.$$

$$\boxed{X(K) = f_1(K) + W_N^K\, f_2(K)} \qquad \text{——— (1)}$$

Where

$F_1(K) = \frac{N}{2}$ Point DFT of $f_1(n)$

$F_2(K) = \frac{N}{2}$ Point DFT of $f_2(n)$.

$F_1(K)$ and $F_2(K)$ are periodic in Nature. So we can write.

$$F_1\left(K+\frac{N}{2}\right) = F_1(K) \text{ and}$$

$$F_2\left(K+\frac{N}{2}\right) = F_2(K).$$

So, eq (1) becomes,

$$X\left(K+\frac{N}{2}\right) = F_1\left(K+\frac{N}{2}\right) + W_N^{K+\frac{N}{2}} F_2\left(K+N/2\right)$$

$$\boxed{X\left(K+\frac{N}{2}\right) = F_1(K) - W_N^K F_2(K)} \quad \text{—(2)} \qquad \because W_N^{K+\frac{N}{2}} = W_N^K.$$

The N-point DFT $X(K)$ can be obtained by combining eq (1) and (2).

$$X(K) = F_1(K) + W_N^K F_2(K)$$

$$X\left(K+\frac{N}{2}\right) = F_1(K) - W_N^K F_2(K)$$

Second stage of Decimation

We know that $f_1(n)$ and $f_2(n)$ are $\frac{N}{2}$ point-sequences.

Now $f_1(n)$ can be splitted into even numbered and odd numbered sampled as follows.

$$\left.\begin{aligned} V_{11}(n) &= f_1(2n) \;;\; n = 0, 1, \ldots \tfrac{N}{4}-1. \\ V_{12}(n) &= f_2(2n+1) \;;\; n = 0, 1, \ldots \tfrac{N}{4}-1 \end{aligned}\right\} \quad \text{—(I)}$$

$V_{21}(n)$

Where, $V_{11}(n)$ is even numbered sequence of $f_1(n)$.

$V_{12}(n)$ is odd numbered sequence of $f_1(n)$.

Similarly $f_2(n)$ can be splitted into even and odd numbered sequences.

$$V_{21}(n) = f_2(2n) \quad ; n = 0, 1, \ldots \frac{N}{4} - 1$$
$$V_{22}(n) = f_2(2n+1) \quad ; n = 0, 1, \ldots N/4 - 1 \Bigg\} \text{ —— II.}$$

$V_{21}(n)$ and $V_{22}(n)$ contain $\frac{N}{4}$ samples each.

equation (I) and II can be written as

$$F_1(K) = V_{11}(K) + W_{N/2}^{K} V_{12}(K) \qquad K = 0, 1, \ldots \frac{N}{4} - 1.$$

$$F_1\left(K + \frac{N}{4}\right) = V_{11}(k) - W_{N/2}^{K} V_{12}(K) \qquad K = 0, 1, \ldots \frac{N}{4} - 1$$

III^{ly} We can write $F_2(K)$ as follows.

$$F_2(K) = V_{21}(K) + W_{N/2}^{K} V_{22}(K) \; ; \; K = 0, 1, \ldots \frac{N}{4} - 1$$

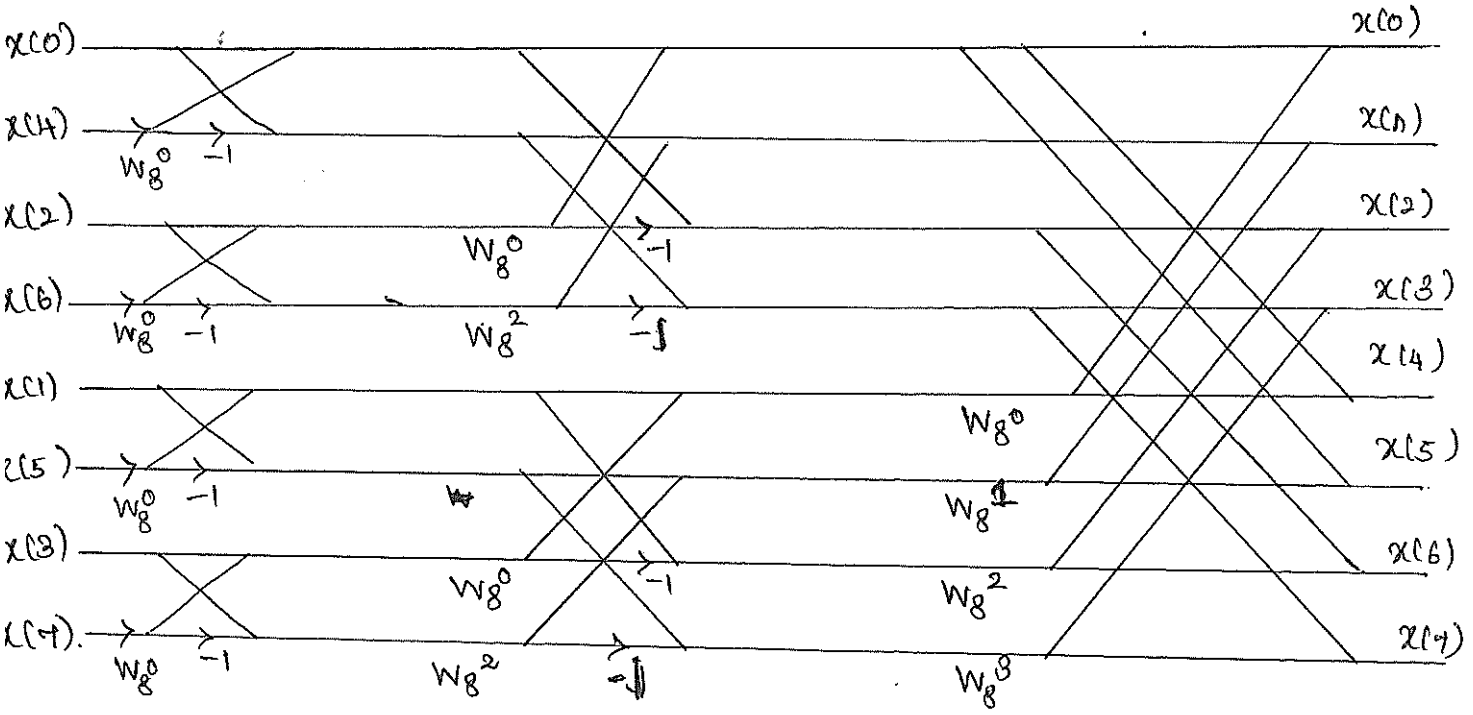
$$F_2\left(K + \frac{N}{4}\right) = V_{21}(K) - W_{N/2}^{K} V_{22}(K) \; ; K = 0, 1, \ldots \frac{N}{4} - 1$$

Where

$V_{21}(K)$ is $N/4$ point DFT of $V_{21}(n)$

$V_{22}(K)$ is $N/4$ Point DFT of $V_{22}(n)$.

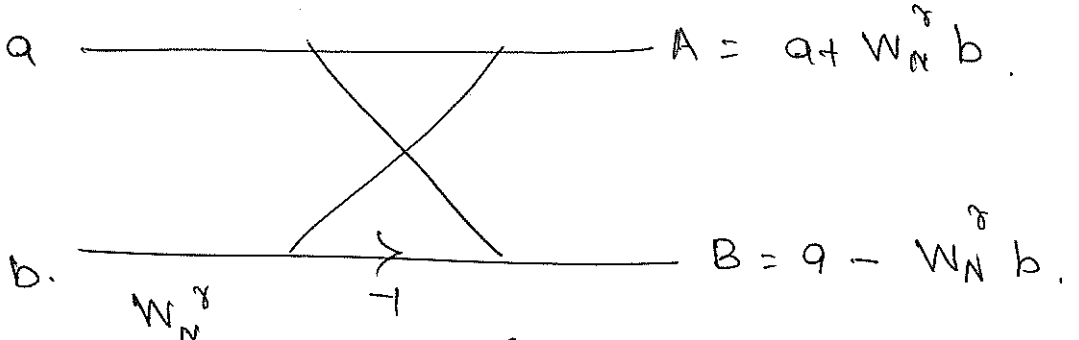
Signal flow graph for N=8.



Bit Reversal,

Decimal	Binary	Bit reversed	decimal.
0	000	000	0
1	001	100	4
2	010	010	2
3	011	110	6
4	100	001	1
5	101	101	5
6	110	011	3
7	111	111	7.

Butterfly Computation



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4) Find the DFT of the sequence $x(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}$ Using DIT-FFT algorithm and DIF-FFT Radix-2 algorithm.

Dec 2013
Dec 2012

Soln:

(a) DIT - FFT algorithm :-

Twiddle factors associated with the Hocographs.

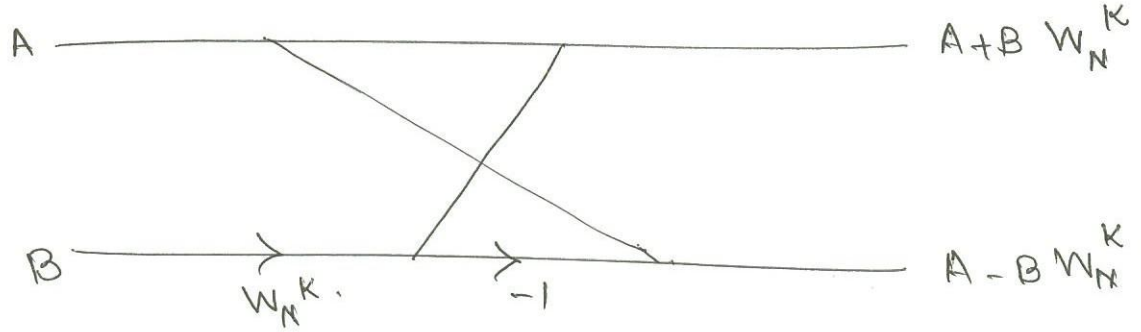
$$W_8^0 = 1$$

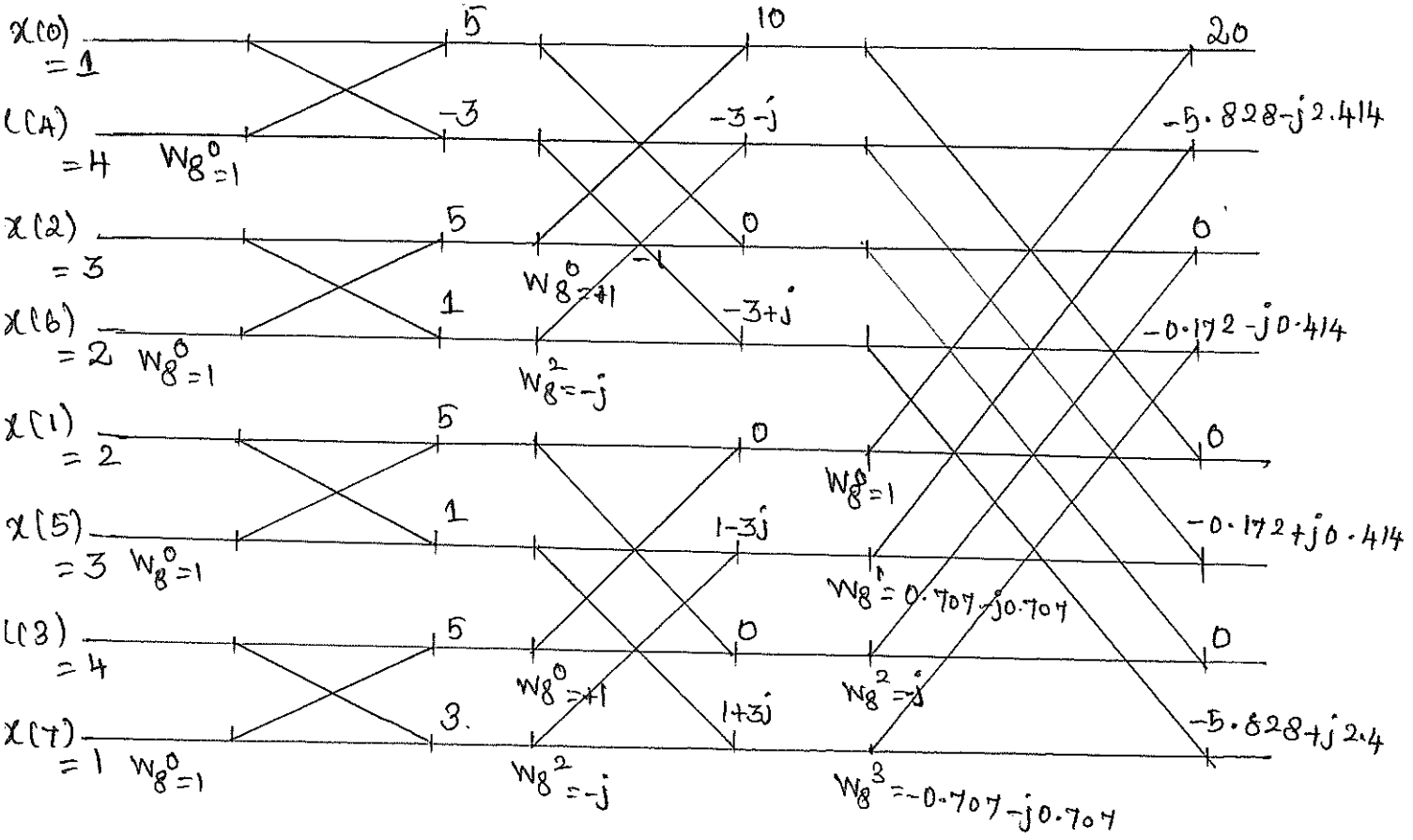
$$W_8^1 = e^{(-j2\pi/8)^1} = 0.707 - j0.707.$$

$$W_8^2 = -j$$

$$W_8^3 = -0.707 - j0.707.$$

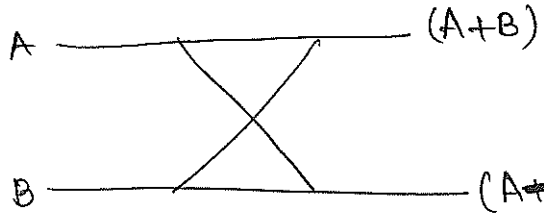
The basic butterfly diagram,

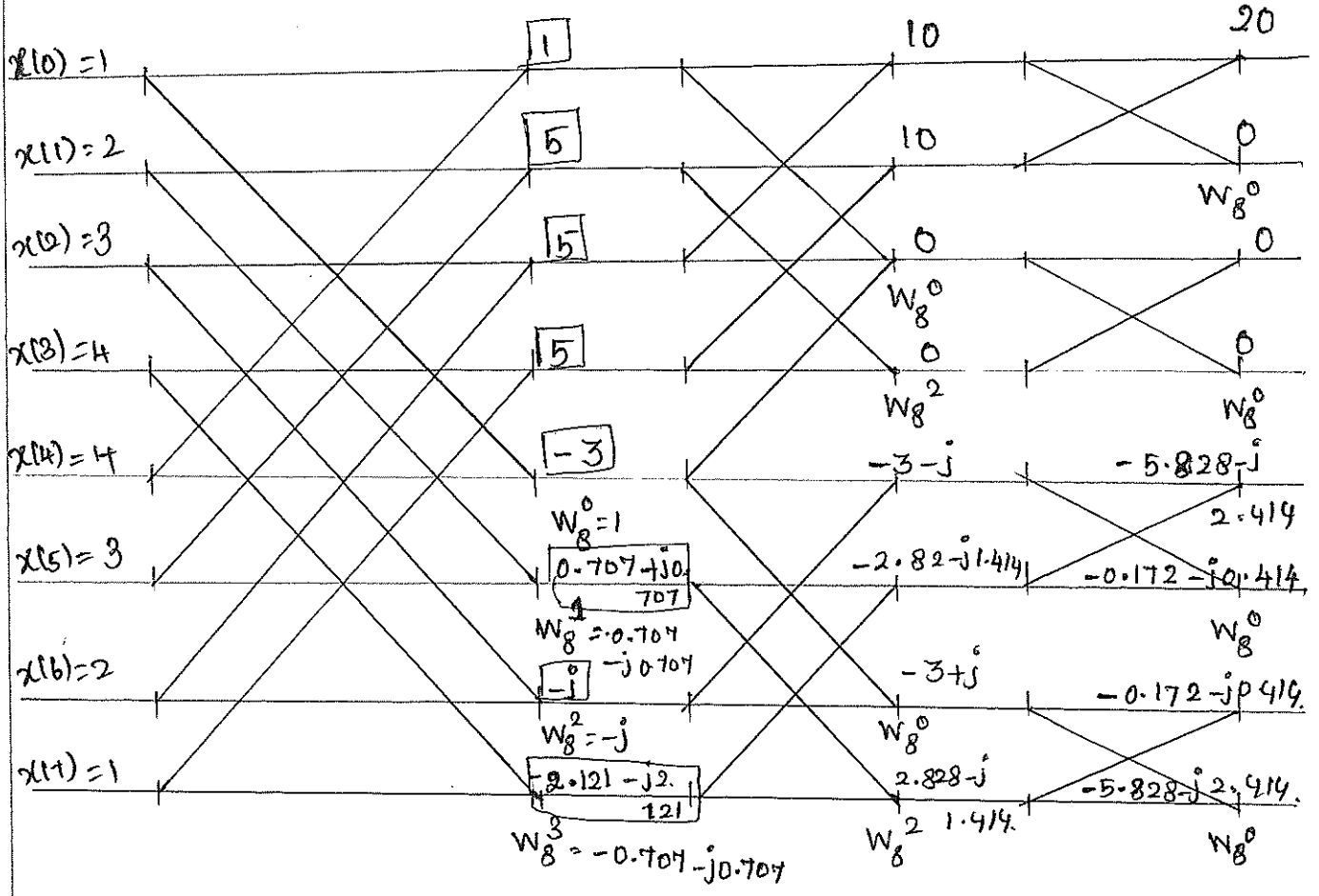




I/p	O/p stage 1	stage 2	stage 3.
1	1+4 = 5	5+5 = 10	10+10 = <u>20</u>
4	1 - 4 = -3	3+(-j) = -3 -j	-3-j+(0.707 -j0.707)(-j -3j) = -5.828 - j2.414.
3	3+2 = 5	5 -5 = 0.	<u>0</u>
2	3-2 = 1	-3-(-j) = -3+j	(-3+j)+(-0.707 - j0.707)(-1+3j) = -0.172 -j0.414.
2	2+3 = 5	5+5 = 0.	10 - 10 = <u>0</u>.
3	2-3 = -1	-1+(-j3) = -1-3j	-3-j-(0.707 -j 0.707)(1-3j) = <u>-0.172 +j0.414</u>
4	4+1 = 5	5 -5 = 0	<u>0</u>
1	4-1 = 3	1-(-3j) = 1+3j	-3+j-(-0.707-j0.707)(-3+j) = -5.828+j2.4.

(b) DIF - FFT algorithm.





i/p	stage 1	2	3 o/p.
1	1+4 = 5	5+5 = 10	10+10 = 20
2	2+3 = 5	5+5 = 10	10 - 10 = 0
3	3+2 = 5	5 - 5 = 0	0
4	4+1 = 5	(5-5) W_8^0 = 0	0
4	-3	-3 - j	-3 - j + (2.82 - j1.414 = -5.828 - j 2.414.

i/p	S_1	S_2	S_3
3	$(2-3)(0.707-j0.707)$ $= 0.707+j0.707$	$(0.707+j0.707+$ $(-2.121-j2.121)$ $= -2.828-j1.414.$	$-3-j+2.828$ $+j1.414$ $= 0.172+j0.414$
2	$(3-2)-j = -j$	$-3(-j) = -3+j$	$-3+j+2.828$ $-j1.414$ $= -0.172-j0.414$
1	$(4-1)(-0.707-j0.707)$ $= -2.121-j2.121$	$2.828-j1.414$	$-5.828+$ $2.414.$

$x(k) = \{20, -5.828, -j2.414, -0.172-j0.414, 0,$
$-0.172+j0.414, 0, -5.828+j2.414\}.$

5). Determine the output of a linear FIR filter whose impulse response $h(n) = \{1, -3, 5\}$ and input signal $x(n) = \{-1, 4, 7, 3, -2, 9, 10, 12, -5, 8\}$. Using (a) Overlap-save Method. APR/May 2015
(b) Overlap-add Method. Dec 2013

(a). Over lap Save Method:

$$x(n) = \{1, 4, 7, 3, -2, 9, 10, 12, -5, 8\}.$$

Divide the input signal into $L = 3$ data sub blocks. Since the length of the impulse response is given by $M = 3$

$$N = L + M - 1 = 3 + 3 - 1 = 5$$

∴ add 2 zeros to the first data subblock.

$x_1(n) = \{0, 0, 1, 4, 7\}$

$x_2(n) = \{4, 7, 3, -2, 9\}$

$x_3(n) = \{-2, 9, 10, 12, -5\}$

$x_4(n) = \{12, -5, 8, 0, 0\}$.

To perform Circular Convolution $x(n)$ and $h(n)$ must be equal, ∴ $h(n) = \{1, -3, 5, 0, 0\}$.

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Perform Circular Convolution on each data subblock.
with impulse response.

$$y_1(n) = x_1(n) \textcircled{N} h(n) = \{0,0,-1,4,7\} \textcircled{N} \{1,-3,5,0,0\}$$

$$\begin{bmatrix} 0 & 7 & 4 & -1 & 0 \\ 0 & 0 & 7 & 4 & -1 \\ -1 & 0 & 0 & 7 & 4 \\ 4 & -1 & 0 & 0 & 7 \\ 7 & 4 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \\ 5 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0-21+20+0+0 \\ 0+0+35+0+0 \\ -1+0+0+0+0 \\ 4+3+0+0+0 \\ 7-12-5+0+0 \end{bmatrix} = \begin{bmatrix} -1 \\ 35 \\ -1 \\ 7 \\ 10 \end{bmatrix}$$

$$y_2(n) = x_2(n) \textcircled{N} h(n) = \{4,7,3,-2,9\} \textcircled{N} \{1,-3,5,0,0\}$$

$$\begin{bmatrix} 4 & 9 & -2 & 3 & 7 \\ 7 & 4 & 9 & -2 & 3 \\ 3 & 7 & 4 & 9 & -2 \\ -2 & 3 & 7 & 4 & 9 \\ 9 & -2 & 3 & 7 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \\ 5 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 4-27-10+0+0 \\ 7-12+45+0+0 \\ 3-21+20+0+0 \\ -2-9+35+0+0 \\ 9+6+15+0+0 \end{bmatrix} = \begin{bmatrix} -33 \\ 40 \\ 2 \\ 24 \\ 30 \end{bmatrix}$$

$$y_3(n) = x_3(n) \textcircled{N} h(n) = \{-2,9,10,12,-5\}$$

$$\textcircled{N} \{1,-3,5,0,0\}$$

$$\begin{bmatrix} -2 & -5 & 12 & 10 & 9 \\ 9 & -2 & -5 & 12 & 10 \\ 10 & 9 & -2 & -5 & 12 \\ 12 & 10 & 9 & -2 & -5 \\ -5 & 12 & 10 & 9 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \\ 5 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -2+15+60+0+0 \\ 9+6-25+0+0 \\ 10-27-10+0+0 \\ 12-30+45+0+0 \\ -5-36+50+0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 73 \\ -10 \\ -27 \\ 27 \\ 9 \end{bmatrix}$$

Page.

$y_4(n) = x_4(n) \textcircled{N} h(n) = \{12,5,8,0,0\} \textcircled{N} \{1,-3,5,0,0\}$

$$\begin{bmatrix} 12 & 0 & 0 & 8 & -5 \\ -5 & 12 & 0 & 0 & 8 \\ 8 & -15 & 12 & 0 & 0 \\ 0 & 8 & -5 & 12 & 0 \\ 0 & 0 & 8 & -5 & 12 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \\ 5 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 12+0+0+0+0 \\ -5-36+0+0+0 \\ 8+15+60+0+0 \\ 0-24-24+0+0 \\ 0+0+40+0+0 \end{bmatrix} = \begin{bmatrix} 12 \\ -41 \\ 83 \\ -49 \\ 40 \end{bmatrix}$$

$y_1(n) =$ | 1 | 35 | -1 | 7 | 10 |

Discard.

$y_2(n) =$ | -33 | 40 | 2 | 24 | 30 |

Discard

$y_3(n) =$ | 73 | -10 | -27 | 27 | 9 |

Discard

$y_4(n) =$ | 12 | -41 | 83 | 49 | 40 |

discard

$\therefore y(n) = \{-1, 7, 10, 2, 24, 30, -27, 27, 9, 83, -49, 40\}$

(b) Overlap add Method:

* Divide the input signal into $L = 3$ data sub blocks
* Since the length of the impulse response is given by $M = 3$ add $M-1 = 2$ zeros to end of each data sub block.

$$N = L + M - 1 = 3 + 2 = 5$$

$x_1(n) = \{-1, 4, 7, 0, 0\}$

$x_2(n) = \{3, -2, 9, 0, 0\}$

$x_3(n) = \{10, 12, -5, 0, 0\}$

$x_4(n) = \{8, 0, 0, 0, 0\}$.

Zero pad the impulse response in order to have to equal data length.

$$h(n) = \{1, -3, 5, 0, 0\}.$$

∴ Perform Circular convolution on each data sub blocks with impulse response.

$$y_1(n) = x_1(n) \textcircled{N} h(n)$$

$$= \{-1, 4, 7, 0, 0\} \textcircled{N} \{1, -3, 5, 0, 0\}$$

$$\begin{bmatrix} -1 & 0 & 0 & 7 & 4 \\ 4 & -1 & 0 & 0 & 7 \\ 7 & 4 & -1 & 0 & 0 \\ 0 & 7 & 4 & -1 & 0 \\ 0 & 0 & 7 & 4 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \\ 5 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1+0+0+0+0 \\ 4+3+0+0+0 \\ 7-12-5+0+0 \\ 0-12+20+0+0 \\ 0+6+35+0+0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ -7 \\ -10 \\ -1 \\ 35 \end{bmatrix}$$

$y_2(n) = x_2(n) \textcircled{N} h_2(n) = \{3,-2,9,0,0\} \textcircled{N} \{1,-3,5,0,0\}$

$$\begin{bmatrix} 3 & 0 & 0 & 9 & -2 \\ -2 & 3 & 0 & 0 & 9 \\ 9 & -2 & 3 & 0 & 0 \\ 9 & 9 & -2 & 3 & 0 \\ 0 & 0 & 9 & -2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \\ 5 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 3+0+0+0+0 \\ -2-9+0+0+0 \\ 9+6+15+0+0 \\ 0-27-10+0+0 \\ 0+0+45+0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ -11 \\ 30 \\ -37 \\ 45 \end{bmatrix}$$

$y_3(n) = x_3(n) \textcircled{N} h_3(n) = \{10,12,-5,0,0\} \textcircled{N} \{1,-3,5,0,0\}$

$$\begin{bmatrix} 10 & 0 & 0 & -5 & 12 \\ 12 & 10 & 0 & 0 & -5 \\ -5 & 12 & 10 & 0 & 0 \\ 0 & -5 & 12 & 10 & 0 \\ 0 & 0 & -5 & 12 & 10 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \\ 5 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 10+0+0+0+0 \\ 12-30+0+0+0 \\ -5-36+50+0+0 \\ 0+15+60+0+0 \\ 0+0-25+0+0 \end{bmatrix} = \begin{bmatrix} 10 \\ -18 \\ 9 \\ 75 \\ -25 \end{bmatrix}$$

$y_4(n) = x_4(n) \textcircled{N} h(n) = \{8,0,0,0,0\} \textcircled{N} \{1,-3,5,0,0\}$

$$\begin{bmatrix} 8 & 0 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 0 & 8 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \\ 5 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 8+0+0+0+0 \\ 0-24+0+0+0 \\ 0+0+40+0+0 \\ 0+0+0+0+0 \\ 0+0+0+0+0 \end{bmatrix} = \begin{bmatrix} 8 \\ -24 \\ 40 \\ 0 \\ 0 \end{bmatrix}$$

$y_1(n)$ | -1 | 7 | -10 | -1 | 35 | 30 | -37 | 45 | 9 | 75 | -25 | 40 | 00

3 -11 (add add)

10 -18 (add)

8 -24 (add)

$y_2(n)$

$y_3(n)$

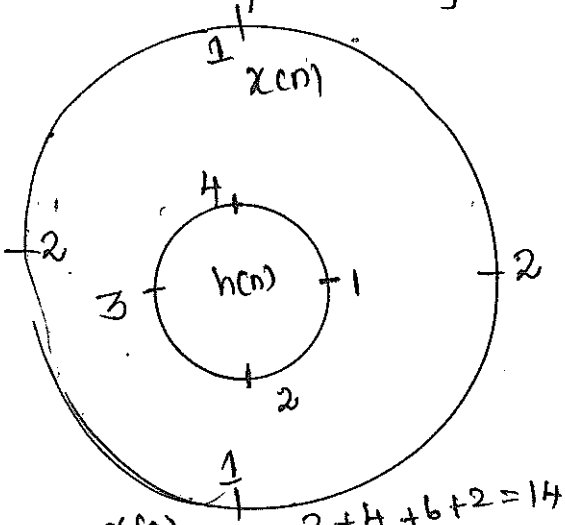
$y_4(n)$

$y(n)$

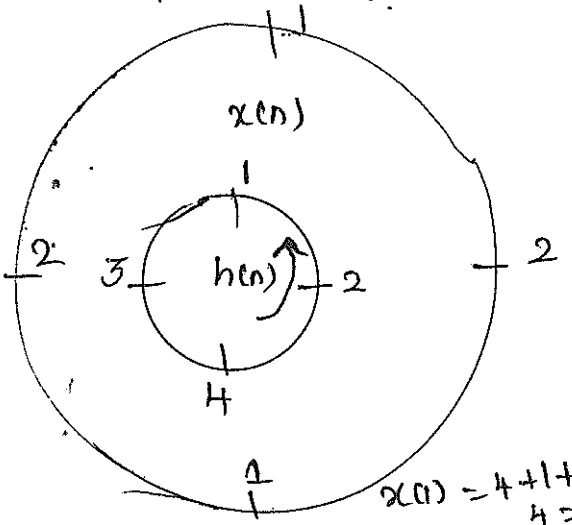
$y(n) = \{-1, 7, -10, 2, 24, 30, -27, 27, 9, 83, -49, 40\}$.

6. Find Circular convolution for the sequence

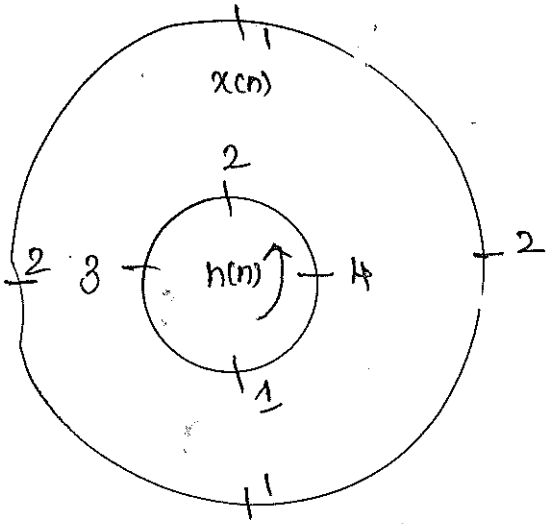
$x(n) = \{2, 1, 2, 1\}$ $h(n) = \{1, 2, 3, 4\}$.



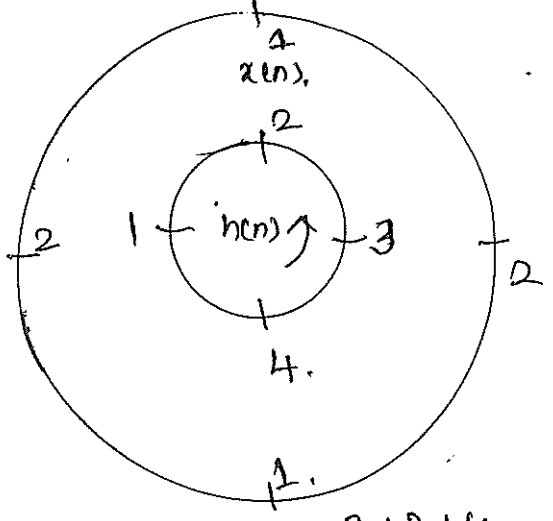
$x(0) = 2+4+6+2 = 14$



$x(1) = 4+1+6+4 = 15$



$x(3) = 8+2+6+1 = 17$



$x(4) = 6+2+2+4$

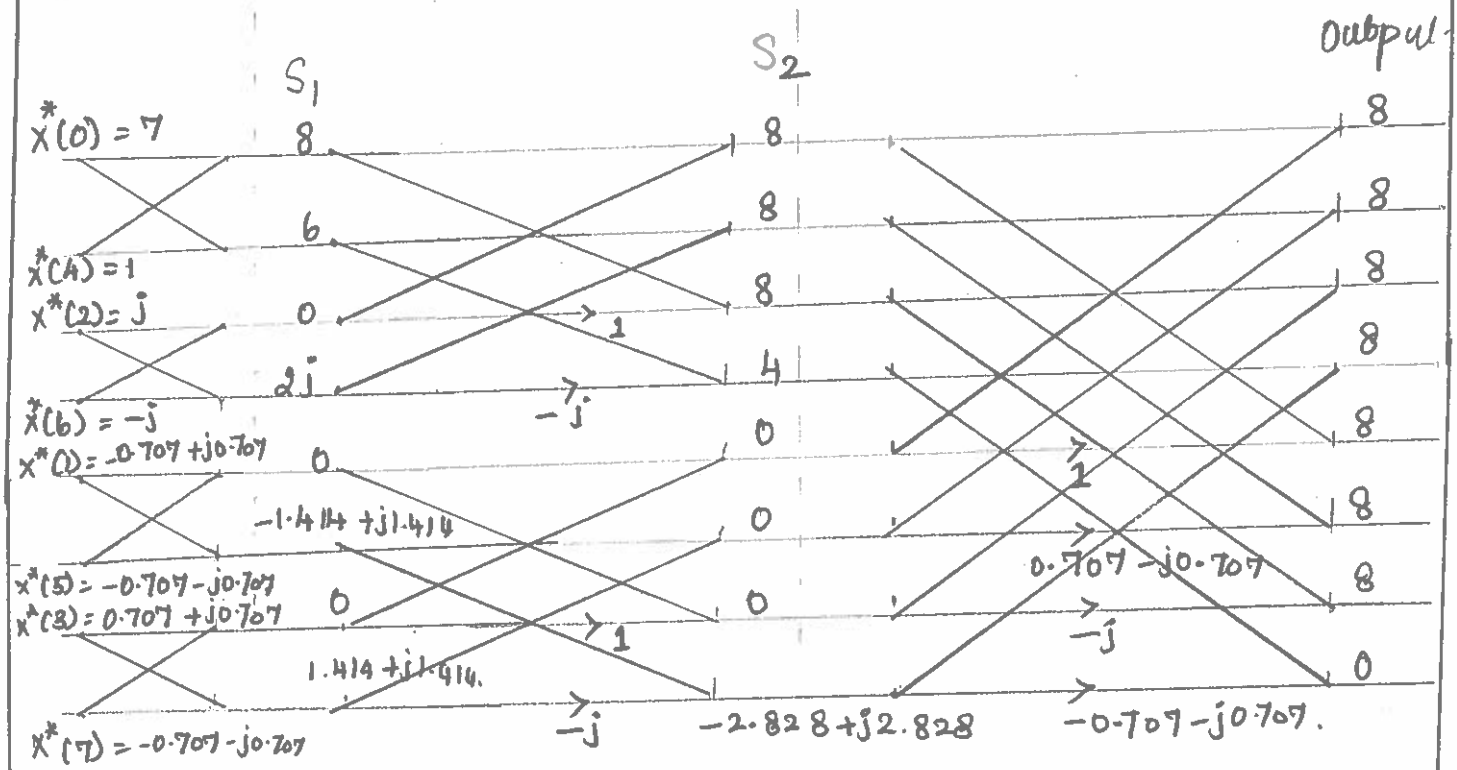
$\therefore x(n) = \{14, 15, 17, 14\} = 14.$

Compute IDFT of the sequence

$X(k) = \{7, -0.707 - j0.707, -j, 0.707 - j0.707, 1, 0.707 + j0.707, j, -.707 + j0.707\}$ Nov|Dec 2016.

using DIT algorithm.

Soln: Take complex conjugate of X(k) and apply bit reversal



Input	S_1	S_2	Output
7	$7+1=8$	$8+0=8$	$8+0=8$
1	$7-1=6$	$6+(-j)(2j)=8$	$8+0(W^1)=8$
j	$j-j=0$	$8-0=8$	$8+0(W^2)=8$
-j	$j-(-j)=2j$	$6-(-j)(2j)=4$	$4+(-0.707-j0.707)(-2.828+j2.828)=8$

$-0.707+j0.707$	$(-0.707+j.707)$ $+(0.707-j0.707)=0$	0	$8-0=8$.
$0.707-j0.707$	$(-0.707+j0.707)$ $-(0.707-j0.707)$	$(-1.414+j1.414)$ $+(-j)(1.414+j1.414$ $=0$	$8-(0(W^1)=8$.
$0.707+j0.707$	$(0.707+j0.707)$ $+(-0.707-j0.707)$ $=0$	0	$8-0(W^2)=8$.
$-0.707-j0.707$	$0.707+j0.707$ $-(-0.707-j0.707)$ $=1.414+j1.414$	$(-1.414+j1.414)$ $-(-j)(1.414+j1.414)$ $=-2.828+j2.828$	$4-(-0.707-j0.707)$ $(-2.828+j2.828)$ $=0$.

$$N(x^1(n)) = \{8, 8, 8, 8, 8, 8, 0\}$$

$$x(n) = \{1, 1, 1, 1, 1, 1, 1, 0\}.$$

Perform the Linear Convolution of finite duration sequence $h(n)=\{1, 2\}$ and $x(n)=\{1, 2, -1, 2, 3, -2, -3, -1, 1, 1, 2, -1\}$.

by overlap save method.

Solution

The linear Convolution of $x(n)$ and $h(n)$ is

$$y(n) = x(n) * h(n)$$

The input sequence can be divided into blocks of data as shown below.

$\rightarrow$ M-1 zero appended.

$x_1 = \{0, 1, 2, -1\}$

3 datas

$x_2 = \{-1, 2, 3, -2\}$

3 new data.

$\rightarrow$ M-1 data from previous block.

$x_3 = \{-2, -3, -1, 1\}$

$x_4 = \{1, 1, 2, -1\}$

$x_5 = \{-1, 0, 0, 0\}$.

To make $h(n) = x(n)$ append 2 zeros.

$\therefore h(n) = \{1, 2, 0, 0\}$.

$\therefore y_1(n) = x_1(n) \textcircled{N} h(n)$

$$= \begin{bmatrix} 1 & 0 & 0 & 2 \\ 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 4 \\ 3 \end{bmatrix}$$

$y_2(n) = x_2(n) \textcircled{N} h(n)$

$$= \begin{bmatrix} 1 & 0 & 0 & 2 \\ 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 3 \\ -2 \end{bmatrix} = \begin{bmatrix} -5 \\ 0 \\ 7 \\ 4 \end{bmatrix}$$

$y_3(n) = x_3(n) \textcircled{N} h(n)$

$$= \begin{bmatrix} 1 & 0 & 0 & 2 \\ 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ -3 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -7 \\ -7 \\ -1 \end{bmatrix}$$

$y_4(n) = x_4(n) \textcircled{N} h(n)$

$$= \begin{bmatrix} 1 & 0 & 0 & 2 \\ 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 4 \\ 3 \end{bmatrix}$$

$y_5(n) = x_5(n) \textcircled{N} h(n)$

$$= \begin{bmatrix} 1 & 0 & 0 & 2 \\ 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ 0 \\ 0 \end{bmatrix}$$

$y_1(n) = \boxed{-2}\ 1\ 4\ -3$

$y_2(n) =$ discard $\boxed{-5}\ 0\ 7\ 4$

$y_3(n) = \boxed{0}\ -7\ -7\ -1$

$y_4(n) = \boxed{-1}\ 3\ 4\ 3$

$y_5(n) = \boxed{-2}$

$y(n) = \ldots \{1, 4, -3, 0, 7, 4, -7, -7, -1, 3, 4, 3, -2, 0, 0\}$

$\therefore y(n) = \{1, 4, -3, 0, 7, 4, -7, -7, -1, 3, 4, 3, 2\}$

In a LTI system the input $x_1(n) = \{1, 2, 3, 4\}$ $h_2(n) = \{1, 1, 2, 1\}$. perform the circular convolution using DFT and IDFT.

Solution

We Know $X_3(K) = X_1(K)\, X_2(K)$

$$X_1(K) = \sum_{n=0}^{N-1} x_1(n)\, e^{-j2\pi kn/N} \qquad K = 0, 1, \ldots N-1$$

Given $x_1(n) = \{1, 1, 2, 1\}$ and $N = 4$.

$$X_1(0) = \sum_{n=0}^{3} x_1(n) = 1+1+2+1 = 5$$

$$X_1(1) = \sum_{n=0}^{3} x_1(n) \cdot e^{-j\pi n/2} = 1 - j + 2 - j = -1$$

$$X_1(2) = \sum_{n=0}^{3} x_1(n)\, e^{-j\pi n} = 1 - 1 + 2 - 1 = \underline{1}$$

$$X_1(3) = \sum_{n=0}^{3} x_1(n)\, e^{-j3\pi n/2} = 1 + j - 2 - j = -1$$

$$X_1(K) = (5, -1, 1, -1)$$

$$X_2(K) = \sum_{n=0}^{N-1} x_2(n)\, e^{-j2\pi nK/N} \qquad K = 0, 1, \ldots N-1$$

$$X_2(0) = \sum_{n=0}^{3} x_2(n) = 1 + 2 + 3 + 4 = 10$$

$$X_2(1) = \sum_{n=0}^{3} x_2(n)\, e^{-j\pi n/2} = 1 + 2(-j) + 3(-1) + 4(j)$$

$$= -2 + j2$$

$$X_2(2) = \sum_{n=0}^{3} x_2(n)\, e^{-j\pi n} = 1+2(-1)+3(1)+4(-1) = -2$$

$$X_2(3) = \sum_{n=0}^{3} x_2(n)\, e^{-j3\pi n/2} = 1+2(j)+3(-1)+4(-j)$$

$$= -2-j2$$

$$X_2(k) = \{10,\ -2+2j,\ -2,\ -2,\ -j2\}.$$

$$X_3(k) = X_1(k)\, X_2(k) = \{50,\ 2-j2,\ -2,\ -2+j2\}$$

$$x_3(n) = \frac{1}{N} \sum_{k=0}^{N-1} X_3(k)\, e^{j2\pi nk/N} \qquad n = 0, 1, \ldots N-1$$

$$x_3(0) = \frac{1}{4} \sum_{k=0}^{3} X_3(k) = \frac{1}{4}(50+2-j2-2+2-j2)$$

$$= 13$$

$$x_3(1) = \frac{1}{4} \sum_{k=0}^{N-1} X_3(k)\, e^{j\pi k/2}$$

$$= \frac{1}{4}\left[50+(2-j2)j+(-2)(-1)+(2+j2)(-j)\right]$$

$$= 14.$$

$$x_3(2) = \frac{1}{4}\left[\sum_{k=0}^{N-1} X_3(k)\, e^{j\pi k}\right]$$

$$= \frac{1}{4}\left[50+(2-j2)j+(-2)(1)+(2+j2)(-1)\right.$$

$$= 11.$$

$$X_8(8) = \frac{1}{4}\left[\sum_{k=0}^{3} x_8(k)\, e^{j3\pi k/2}\right]$$

$$= \frac{1}{4}\left[50 + (2-j2)(-j) + (-2)(-1) + (2+j2)(j)\right]$$

$$= 12.$$

$$x_8(n) = \{13, 14, 11, 12\}.$$

UNIT : II

IIR FILTER DESIGN.

PART_A (2 Marks).

1. Distinguish between Butterworth and Chebyshev filter.

Butterworth filter	Chebyshev filter
* Magnitude response decreses monotonically as the frequency Ω increases from 0 to ∞.	Magnitude response exhibits ripple in the passband and monotonically decreasing the stopband.
* Transition band is more	* Transition band is less.
* Poles are lie on a circle	* Poles are lie on an ellipse
* Number of poles are more	* Number of poles are less.

2. What is prewarping?

* In IIR design using bilinear transformation the conversion of the specified digital frequencies to analog frequencies is called prewraping.

* It is necessary to eliminate the effect of wraping on amplitude response.

$$\Omega = \frac{2}{T} \tan \frac{\omega}{2}.$$

$-0.707+j0.707$	$(-0.707+j.707)$ $+(0.707-j0.707)=0$	0	$8-0=8$.
$0.707-j0.707$	$(-0.707+j0.707)$ $-(0.707-j0.707)$	$(-1.414+j1.414)$ $+(-j)(1.414+j1.414$ $=0$	$8-(0(W^1)=8$.
$0.707+j0.707$	$(0.707+j0.707)$ $+(-0.707-j0.707)$ $=0$	0	$8-0(W^2)=8$.
$-0.707-j0.707$	$0.707+j0.707$ $-(-0.707-j0.707)$ $=1.414+j1.414$	$(-1.414+j1.414)$ $-(-j)(1.414+j1.414)$ $=-2.828+j2.828$	$4-(-0.707-j0.707)$ $(-2.828+j2.828)$ $=0$.

$$N(x^1(n)) = \{8, 8, 8, 8, 8, 8, 0\}$$

$$x(n) = \{1, 1, 1, 1, 1, 1, 1, 0\}.$$

Perform the Linear Convolution of finite duration sequence $h(n)=\{1, 2\}$ and $x(n)=\{1, 2, -1, 2, 3, -2, -3, -1, 1, 1, 2, -1\}$.

by overlap save method.

Solution

The linear Convolution of $x(n)$ and $h(n)$ is

$$y(n) = x(n) * h(n)$$

The input sequence can be divided into blocks of data as shown below.

$\rightarrow$ M-1 zero appended.

$x_1 = \{0, 1, 2, -1\}$

3 datas

$x_2 = \{-1, 2, 3, -2\}$

3 new data.

$\rightarrow$ M-1 data from previous block.

$x_3 = \{-2, -3, -1, 1\}$

$x_4 = \{1, 1, 2, -1\}$

$x_5 = \{-1, 0, 0, 0\}$.

To make $h(n) = x(n)$ append 2 zeros.

$\therefore h(n) = \{1, 2, 0, 0\}$.

$\therefore y_1(n) = x_1(n) \textcircled{N} h(n)$

$$= \begin{bmatrix} 1 & 0 & 0 & 2 \\ 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 4 \\ 3 \end{bmatrix}$$

$y_2(n) = x_2(n) \textcircled{N} h(n)$

$$= \begin{bmatrix} 1 & 0 & 0 & 2 \\ 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \\ 3 \\ -2 \end{bmatrix} = \begin{bmatrix} -5 \\ 0 \\ 7 \\ 4 \end{bmatrix}$$

$y_3(n) = x_3(n) \textcircled{N} h(n)$

$$= \begin{bmatrix} 1 & 0 & 0 & 2 \\ 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ -3 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -7 \\ -7 \\ -1 \end{bmatrix}$$

$y_4(n) = x_4(n) \textcircled{N} h(n)$

$$= \begin{bmatrix} 1 & 0 & 0 & 2 \\ 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 4 \\ 3 \end{bmatrix}$$

$y_5(n) = x_5(n) \textcircled{N} h(n)$

$$= \begin{bmatrix} 1 & 0 & 0 & 2 \\ 2 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ 0 \\ 0 \end{bmatrix}$$

$y_1(n) = \boxed{-2}\ 1\ 4\ -3$

$y_2(n) =$ discard $\boxed{-5}\ 0\ 7\ 4$

$y_3(n) = \boxed{0}\ -7\ -7\ -1$

$y_4(n) = \boxed{-1}\ 3\ 4\ 3$

$y_5(n) =$ $\boxed{-1}$

$y(n) = \ldots \{1, 4, -3, 0, 7, 4, -7, -7, -1, 3, 4, 3, -2, 0, 0\}$

$\therefore y(n) = \{1, 4, -3, 0, 7, 4, -7, -7, -1, 3, 4, 3, 2\}$

In a LTI system the input $x_1(n) = \{1, 2, 3, 4\}$ $h_2(n) = \{1, 1, 2, 1\}$. perform the circular convolution using DFT and IDFT.

Solution

We Know $X_3(K) = X_1(K)\, X_2(K)$

$$X_1(K) = \sum_{n=0}^{N-1} x_1(n)\, e^{-j2\pi kn/N} \qquad K = 0, 1, \ldots N-1$$

Given $x_1(n) = \{1, 1, 2, 1\}$ and $N = 4$.

$$X_1(0) = \sum_{n=0}^{3} x_1(n) = 1+1+2+1 = 5$$

$$X_1(1) = \sum_{n=0}^{3} x_1(n) \cdot e^{-j\pi n/2} = 1 - j + 2 - j = -1$$

$$X_1(2) = \sum_{n=0}^{3} x_1(n)\, e^{-j\pi n} = 1 - 1 + 2 - 1 = \underline{1}$$

$$X_1(3) = \sum_{n=0}^{3} x_1(n)\, e^{-j3\pi n/2} = 1 + j - 2 - j = -1$$

$$X_1(K) = (5, -1, 1, -1)$$

$$X_2(K) = \sum_{n=0}^{N-1} x_2(n)\, e^{-j2\pi nK/N} \qquad K = 0, 1, \ldots N-1$$

$$X_2(0) = \sum_{n=0}^{3} x_2(n) = 1 + 2 + 3 + 4 = 10$$

$$X_2(1) = \sum_{n=0}^{3} x_2(n)\, e^{-j\pi n/2} = 1 + 2(-j) + 3(-1) + 4(j)$$

$$= -2 + j2$$

$$X_2(2) = \sum_{n=0}^{3} x_2(n)\, e^{-j\pi n} = 1+2(-1)+3(1)+4(-1) = -2$$

$$X_2(3) = \sum_{n=0}^{3} x_2(n)\, e^{-j3\pi n/2} = 1+2(j)+3(-1)+4(-j)$$

$$= -2-j2$$

$$X_2(k) = \{10,\ -2+2j,\ -2,\ -2,\ -j2\}.$$

$$X_3(k) = X_1(k)\, X_2(k) = \{50,\ 2-j2,\ -2,\ -2+j2\}$$

$$x_3(n) = \frac{1}{N} \sum_{k=0}^{N-1} X_3(k)\, e^{j2\pi nk/N} \qquad n = 0, 1, \ldots N-1$$

$$x_3(0) = \frac{1}{4} \sum_{k=0}^{3} X_3(k) = \frac{1}{4}(50+2-j2-2+2-j2)$$

$$= 13$$

$$x_3(1) = \frac{1}{4} \sum_{k=0}^{N-1} X_3(k)\, e^{j\pi k/2}$$

$$= \frac{1}{4}\left[50+(2-j2)j+(-2)(-1)+(2+j2)(-j)\right]$$

$$= 14.$$

$$x_3(2) = \frac{1}{4}\left[\sum_{k=0}^{N-1} X_3(k)\, e^{j\pi k}\right]$$

$$= \frac{1}{4}\left[50+(2-j2)j+(-2)(1)+(2+j2)(-1)\right.$$

$$= 11.$$

$$X_8(8) = \frac{1}{4}\left[\sum_{k=0}^{3} x_8(k)\, e^{j3\pi k/2}\right]$$

$$= \frac{1}{4}\left[50 + (2-j2)(-j) + (-2)(-1) + (2+j2)(j)\right]$$

$$= 12.$$

$$x_8(n) = \{13, 14, 11, 12\}.$$

3. What is bilinear transformation? What are the main advantages and disadvantages of this technique?

* The bilinear transformation is a mapping that transforms the left half of s plane in to the unit circle in the z plane only once, thus avoiding aliasing of frequency components.

* The maping from the s-plane to z-plane in bilinear transformation is,

$$s = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$$

4. Why the butterworth response is called maximally flat response? Dec 13

* Magnitude response of the butterworth filter decreases monotonically as the frequency Ω increases from 0 to ∞.

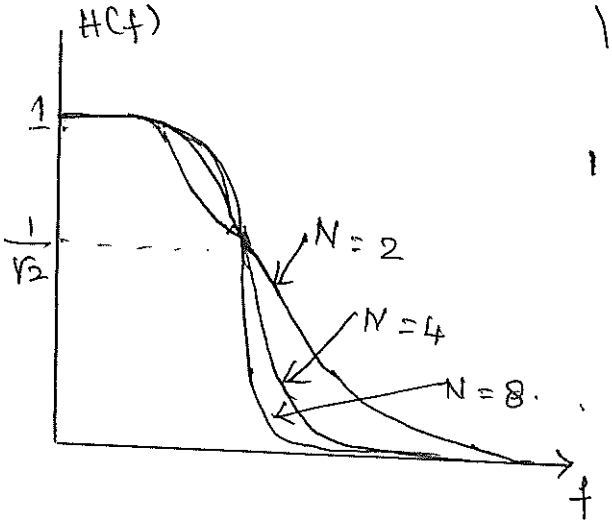
* So the magnitude response is maximally flat at the orgin. For this reason Butterworth filter response is called a maximally flat response.

5. What is frequency wrapping? Dec 12 (8M)

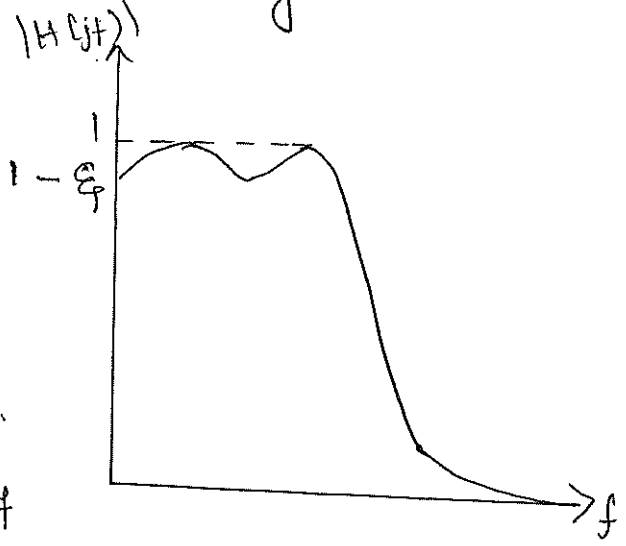
In Bilinear Transformation, the relation between analog and digital frequencies is non linear. When the S-plane is mapped into z-plane using Bilinear transformation, the non linear relationship introduces distortion in frequency axis, which is called frequency wrapping.

6. Draw the response curve for Butterworth, Chebyshev and elliptic filters. APR/May 2015

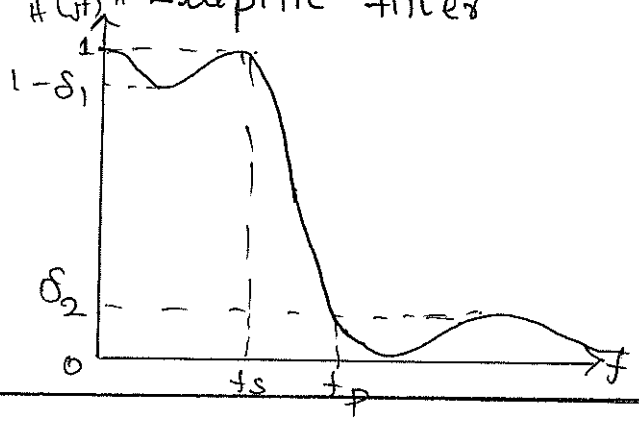
* Butter Worth.



* Chebyshev filter



* Elliptic filter



7. What is the advantage of direct - II realization When compared to direct - I realization?

* In direct II realization, Number of memory locations required is less than the direct form I realization
* same delay is used for input and output.

8. What are the disadvantages of direct form realization? Dec/2013

* It is extremely sensitive to parameter quantization
* When the order of the filter N is large a small change in filter coefficient due to parameter quantization, results large change in the location of the poles and zeros of the system.

9. Mention the advantage of Cascade realization?

* Quantization errors can be minimized if we realize an LTI system in cascade form.

10. Given the steps in the design of a digital filter from analog filter

* Bilinear Transform : substitute $s = \frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]$ in $H(s)$ to convert $H(z)$.

* In Impulse Invariant Method substitute the following in $H(s)$ $H(z) = \frac{C_k}{1-e^{p_k T} z^{-1}}$

UNIT - II

PART - B

1. Obtain the direct form I and II, Cascade, parallel form for the given system function

$$y(n) = -0.1\,y(n-1) + 0.2\,y(n-2) + 3\,x(n) + 6.6\,x(n-1) + 0.6\,x(n-2)$$

Apr/May 2015
June 13.
Dec 12

Soln:

$$y(n) = -0.1\,y(n-1) + 0.2\,y(n-2) + 3\,x(n) + 6.6\,x(n-1) + 0.6\,x(n-2)$$

Take Z transform

$$Y(z) = -0.1\,z^{-1}\,Y(z) + 0.2\,z^{-2}\,Y(z) + 3\,X(z) + 6.6\,z^{-1}\,X(z) + 0.6\,z^{-2}\,X(z)$$

$$Y(z) + 0.1\,z^{-1}\,Y(z) + 0.2\,z^{-2}\,Y(z) = 3\,X(z) + 6.6\,z^{-1}\,X(z) + 0.6\,z^{-2}\,X(z)$$

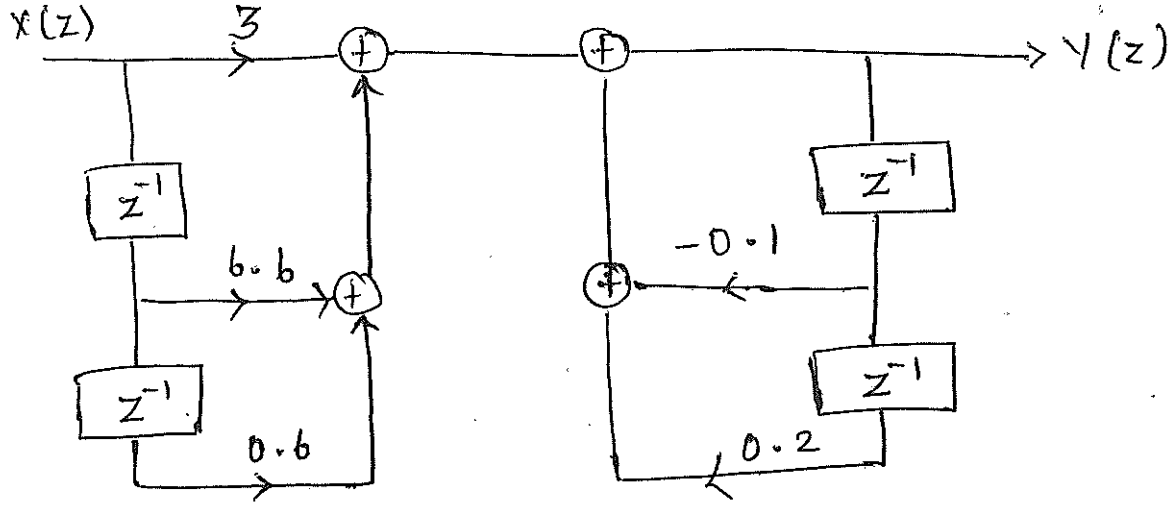
$$Y(z)\left[1 + 0.1\,z^{-1} - 0.2\,z^{-2}\right] = X(z)\left[3 + 6.6\,z^{-1} + 0.6\,z^{-2}\right]$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{3 + 6.6\,z^{-1} + 0.6\,z^{-2}}{1 + 0.1\,z^{-1} - 0.2\,z^{-2}}$$

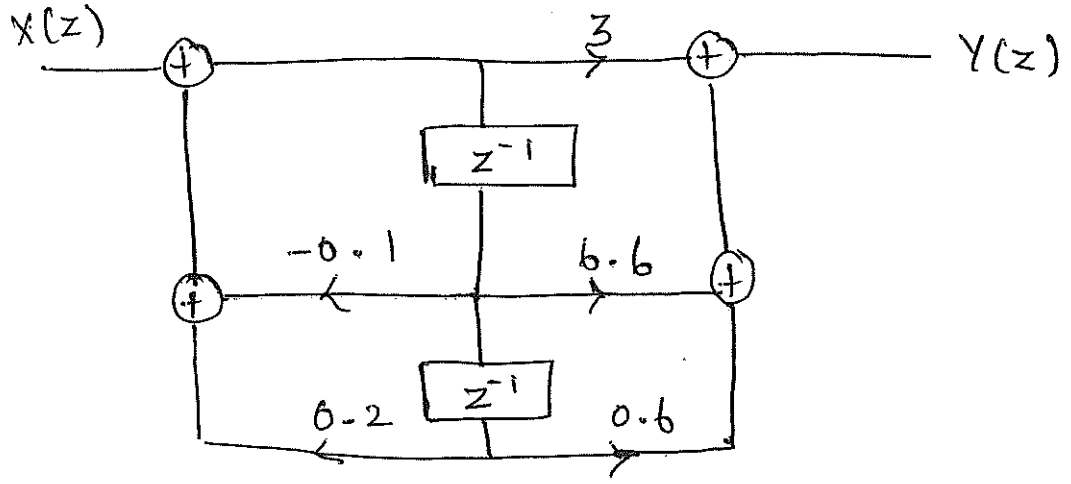
Here, $b_0 = 3 \quad b_1 = 6.6 \quad b_2 = 0.6$

$a_1 = 0.1 \quad a_2 = -0.2$

Direct form I.



Direct form II



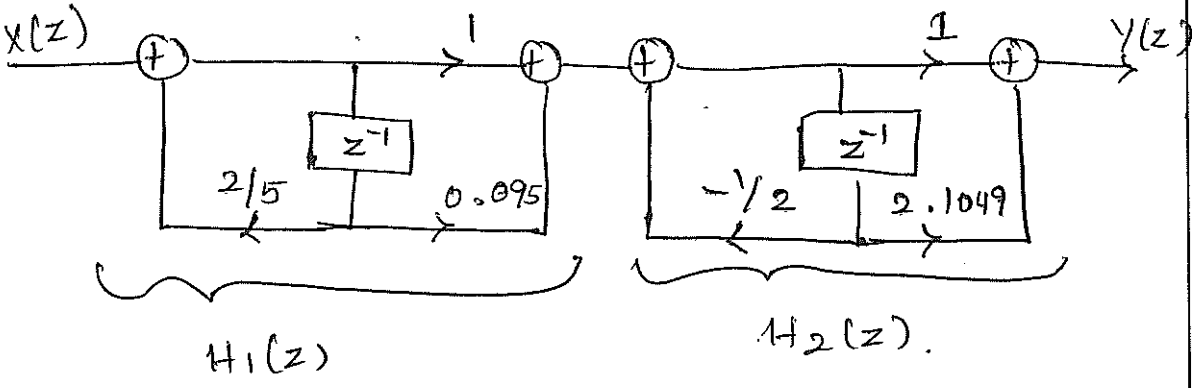
Cascade form:-

$$H(z) = \frac{3 + 6.6z^{-1} + 0.6z^{-2}}{1 + 0.1z^{-1} - 0.2z^{-2}}$$

$$H(z) = \frac{(1 + 0.095z^{-1})(1 + 2.1049z^{-1})}{(1 - \frac{2}{5}z^{-1})(1 + \frac{1}{2}z^{-1})}$$

Let- $H_1(z) = \dfrac{1 + 0.095\ z^{-1}}{1 - \frac{2}{5} z^{-1}}$

$$H_2(z) = \frac{1 + 2.1049 z^{-1}}{1 + \frac{1}{2} z^{-1}}$$



Parallel form.

$$H(z) = \frac{3 + 6.6 z^{-1} + 0.6 z^{-2}}{1 + 0.1 z^{-1} - 0.2 z^{-2}}$$

Multiply and Divide by z^2

$$H(z) = \frac{3z^2 + 6.6z + 0.6}{z^2 + 0.1 z^1 - 0.2}$$

$$H(z) = \frac{3z^2 + 6.6z + 0.6}{(z - \frac{2}{5})(z + \frac{1}{2})}$$

$$\frac{H(z)}{z} = \frac{3z^2 + 6.6z + 0.6}{z(z - \frac{2}{5})(z + \frac{1}{2})}$$

Pa

By partial fraction

$$= \frac{A}{z} + \frac{B}{z-2/5} + \frac{C}{z+1/2}.$$

$$A = \frac{3z^2 + 6.6z + 0.6}{z(z-2/5)(z+1/2)} \cdot z \Big/ z=0$$

$$= 0.6 / -2/10$$

$$\boxed{A = -3}$$

$$B = (z-\tfrac{2}{5}) \cdot \frac{3z^2 + 6.6z + 0.6}{z(z-2/5)(z+1/2)} \Big/ z = 2/5$$

$$= \frac{\frac{2}{25} + \frac{13.2}{5} + 0.6}{2/25\,(2/5 + 1/2)}$$

$$= \frac{\frac{12 + 66 + 15}{25}}{2/5\,(9/10)} = \frac{93}{25} \times \frac{25}{9}.$$

$$\boxed{B = 10.33}$$

$$C = (z+1/2)\,\frac{3z^2 + 6.6z + 0.6}{z(z-\frac{2}{5})(z+1/2)} \Big/ z \to 1/2.$$

$$= \frac{3/4 - \frac{6.6}{2} + 0.6}{-1/2\left(-1/2 - 2/5\right)}$$

$$= \frac{\frac{3 - 13.2 + 2.4}{4}}{-1/2\left(-9/10\right)}$$

$$= -\frac{7.8}{4} \times \frac{20}{9}$$

$$\boxed{C = -4.33}$$

$$\frac{H(z)}{z} = \frac{-3}{z} + \frac{10.33}{\left(z - \frac{2}{5}\right)} - \frac{4.33}{\left(z + 1/2\right)}$$

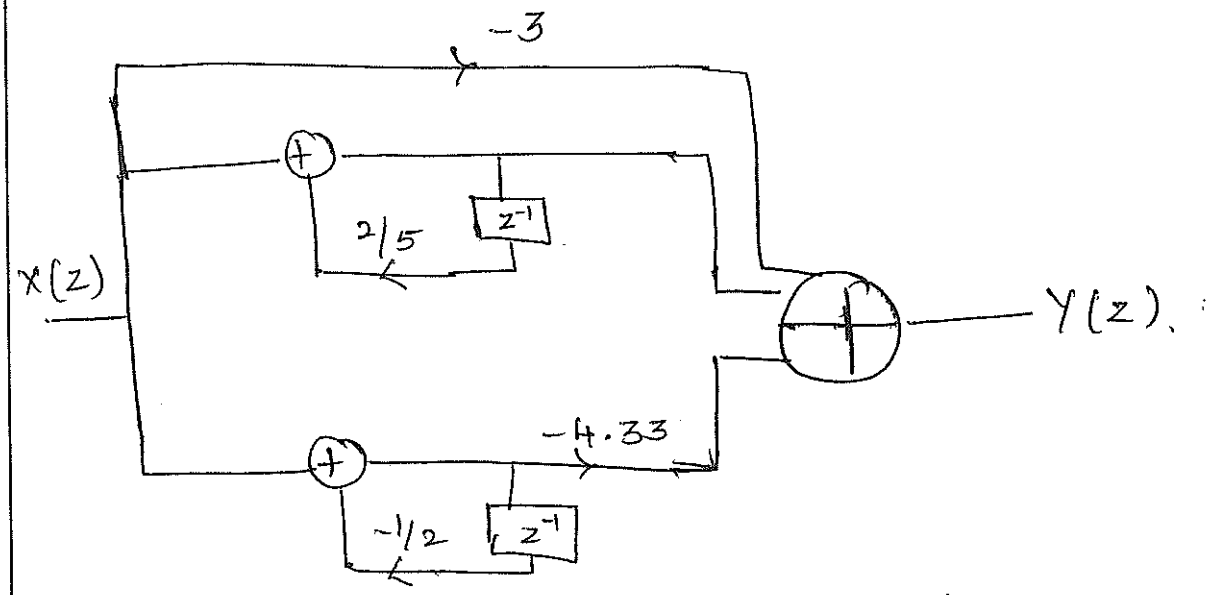
$$H(z) = -3 + \frac{10.33z}{z - 2/5} - \frac{4.33z}{z + 1/2}$$

$$H(z) = -3 + \frac{10.33}{1 - \frac{2}{5}z^{-1}} - \frac{4.33}{1 + 1/2\,z^{-1}}$$

$$C = -3$$

$$H_1(z) = \frac{10.33}{1 - 2/5\,z^{-1}}$$

$$H_2(z) = \frac{-4.33}{1 + 1/2\,z^{-1}}$$



2(i) Convert the analog filter into digital filter whose system function is.

$$H(s) = \frac{s+0.2}{(s+0.2)^2+9}, \qquad 1T = 1\text{sec}.$$

Soln:

$$H(s) = \frac{s+9}{(s+9)^2+b^2} \longrightarrow H(z) = \frac{1 - e^{-9T}(\cos bT)\, z^{-1}}{1 - 2e^{-9T}(\cos bT)\, z^{-1} + e^{-29T} z^{-2}}$$

$T = 1\text{sec}.$

$$H(z) = \frac{1 - e^{-0.2T}(\cos 3T)\, z^{-1}}{(1 - 2e^{-0.2T}(\cos 3T)\, z^{-1} + e^{-0.4T} z^{-2})}$$

$$H(z) = \frac{1 - (0.8187)(-0.99)\, z^{-1}}{1 - 2(0.8187)(-0.99)\, z^{-1} + 0.6703\, z^{-2}}$$

Thus:

$$H(z) = \frac{1 + (0.8105)\, z^{-1}}{1 + 1.6210\, z^{-1} + 0.6708 z^{-1}}$$

2(ii) Convert the given analog transfer function into equivalent digital transfer function using impulse invariant transformation Technique.

June 2013
Dec 12

$$H(s) = \frac{s+0.1}{(s+0.1)^2+9}$$

Soln

Zeros

$s = 0.1$

Poles

$s = -0.1 \pm j3$

$$s^2 + 2\times 0.1 s + (0.1)^2 + 9$$

$$s^2 + 0.2s + 9$$

$$= \frac{-0.2 \pm \sqrt{(0.2)^2 - 4\times 9}}{2}$$

$$\boxed{s = -0.1 \pm j3}$$

$$H(s) = \frac{s+0.1}{(s+0.1-j3)(s+0.1+j3)}$$

$$= \frac{A}{(s+0.1-j3)} + \frac{B}{(s+0.1+j3)}$$

Solving A and B

$A = 1/2$; $B = 1/2$.

$$A = \left[(s+0.1-j3)\frac{s+0.1}{(s+0.1-j3)(s+0.1+j3)} \right]_{s=-0.1+j3}$$

$$A = \frac{1}{2} + 0j \quad \text{and} \quad B = \frac{1}{2} - 0j$$

$$\boxed{A = \frac{1}{2} \quad B = \frac{1}{2}}$$

$$H(s) = \frac{1/2}{(s+0.1-j3)} + \frac{1/2}{(s+0.1+j3)}$$

Using the transformation relation

$$\sum_{i=1}^{N} \frac{1}{(s-p_i)} \longrightarrow \sum_{i=1}^{N} \frac{1}{1-e^{p_i T} z^{-1}}$$

$$H(z) = \frac{1/2}{1-e^{(-0.1+j3)T} z^{-1}} + \frac{1/2}{1-e^{(-0.1-j3)T} z^{-1}}$$

$$H(z) = \frac{\frac{1}{2}\left[1-e^{(-0.1-j3)T} z^{-1}\right] + \frac{1}{2}\left[1-e^{(-0.1+j3)T} z^{-1}\right]}{\left[1-e^{(-0.1+j3)T} z^{-1}\right]\left[1-e^{(-0.1-j3)T} z^{-1}\right]}$$

$$H(z) = \frac{1-e^{-0.1T}\left[\frac{e^{j3T}+e^{-j3T}}{2}\right] z^{-1}}{1-2e^{-0.1T}\left[\frac{e^{j3T}+e^{-j3T}}{2}\right] z^{-1} + e^{-0.2T} z^{-2}}$$

$$H(z) = \frac{1-e^{-0.1T}\cos 3T\, z^{-1}}{1-e^{-0.1T}(2\cos 3T) z^{-1} + e^{-0.2T} z^{-2}}$$

Let us assume $T = 1$ sec/sample.

$$H(z) = \frac{1 - e^{-0.1}\cos 3 z^{-1}}{1 - e^{-0.1} 2\cos 3 z^{-1} + e^{-0.2} z^{-2}}$$

$$\boxed{H(z) = \frac{1 + 0.895 z^{-1}}{1 + 1.791 z^{-1} + 0.818 z^{-2}}}$$

3. Apply Bilinear transformation to $H(s) = \frac{2}{(s+1)(s+2)}$ with $T = 1$ sec. and find $H(z)$.

APR|May 2015

Apr|may 2015.
June 13.

Soln:

By the definition of Bilinear Transformation

$$H(z) = H(s) \Big| \; s \to \frac{2}{T}\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right)$$

$$H(z) = \frac{2}{\left(\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 1\right)\left(\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 2\right)}$$

$$= \frac{2}{\left[\frac{2(1-z^{-1}) + (1+z^{-1})}{1+z^{-1}}\right]\left[\frac{2(1-z^{-1}) + 2(1+z^{-1})}{1+z^{-1}}\right]}$$

$$H(z) = \frac{0.166\left(1+z^{-1}\right)^2}{\left(1-0.33\,z^{-1}\right)}$$

4) Design a Butterworth filter using (i) Impulse Invariance method (ii) Bilinear Transformation

Specification.

Apr/May 2015
Dec 2013
June 13
Dec 12

$$0.8 \le |H(e^{j\omega})| \le 1 \qquad 0 \le \omega \le 0.2\pi$$

$$|H(e^{j\omega})| \le 0.2 \qquad 0.6\pi \le \omega \le \pi$$

Soln:

$$\omega_p = 0.2\pi \quad \omega_s = 0.6\pi \quad \varepsilon_p = 0.8 \quad \delta = 0.2$$

(i) Bilinear Transformation Method:-

The prewrapping analog frequency is,

$$\Omega_s = \frac{2}{T}\tan\left(\frac{\omega_s}{2}\right) = \frac{2}{1}\tan\left(\frac{0.6\pi}{2}\right)$$

$$\boxed{\Omega_s = 2.752}$$

$$\Omega_p = \frac{2}{T}\tan\left(\frac{\omega_p}{2}\right) = \frac{2}{1}\tan\left(\frac{0.2\pi}{2}\right)$$

$$\boxed{\Omega_p = 0.6498}$$

From the given specification

$$\frac{1}{\sqrt{1+\varepsilon^2}} = 0.8 \qquad \frac{1}{1+\varepsilon^2} = 0.8^2$$

$$\varepsilon = 0.75$$

and $$\frac{1}{\sqrt{1+d^2}} = 0.2$$

$$\boxed{d = 4.899}$$

To find order of the filter

$$N \geq \frac{\log(d/\varepsilon)}{\log\left(\frac{\Omega_s}{\Omega_p}\right)}$$

$$N \geq \frac{\log\left(\frac{4.899}{0.75}\right)}{\log\left(\frac{2.752}{0.6496}\right)}$$

$$\geq 1.3.$$

$$\boxed{N = 2}$$

for $N = 2$ the analog transfer function is,

$$H(s) = \frac{1}{s^2+\sqrt{2}s+1}$$

To find cut off frequency

$$\Omega_c = \frac{\Omega_p}{(\varepsilon)^{1/N}}$$

$$= \frac{0.6498}{(0.75)^{1/2}}$$

$$\boxed{\Omega_c = 0.75}$$

To find transfer function for given Ω_c

$$H_a(s) = H(s) \;/\; s \rightarrow s/\Omega_c$$

$$= \frac{1}{\left(\frac{s}{0.75}\right)^2 + \sqrt{2}\left(\frac{s}{0.75}\right) + 1}$$

$$= \frac{1}{\frac{s^2}{(0.75)^2} + \frac{\sqrt{2}s}{(0.75)^2} + 1}$$

$$\boxed{H_a(s) = \frac{0.5625}{s^2 + 1.06s + 0.5625}}$$

To find $H(z)$

for Bilinear transformation,

$$s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) \quad ; T = 1 \text{ sec.}$$

$$H(z) = \frac{0.5625}{\left[\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right]^2 + 1.06 \times \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 0.5625}$$

$$H(z) = \frac{0.5625\,(1+z^{-1})^2}{4\,(1-z^{-1})^2 + 2.12\,(1-z^{-2}) + 0.5625(1+z^{-1})^2}$$

$$H(z) = \frac{0.5625\,(1+z^{-1})^2}{2.44z^{-2} - 6.875z^{-1} + 6.6825}$$

Impulse Invariant Method:

Given $\omega_s = 0.6\pi$ $\omega_p = 0.2\pi$

$\epsilon_p = 0.75$ $A = 4.898$

The analog frequency are

$$\Omega_s = \frac{\omega_s}{T} = 0.6\pi$$

$$\Omega_p = \frac{\omega_p}{T} = 0.2\pi$$

To find the order of the filter

$$N \geq \frac{\log(A/\epsilon)}{\log\left(\frac{\Omega_s}{\Omega_p}\right)}$$

$$\geq \frac{\log(4.89/0.75)}{\log\left(\frac{0.6\pi}{0.2\pi}\right)}$$

$N \geq 1.71$

$$\boxed{N = 2}$$

for $N=2$ the transfer function of the butterworth filter is

$$H(s) = \frac{1}{s^2+\sqrt{2}s+1}$$

To find cut-off frequency

$$\Omega_c = \frac{\Omega_p}{(\xi)^{1/N}} = \frac{0.2\pi}{(0.75)^2}$$

$$\Omega_c = 0.28\pi$$

for the above cut-off frequency, the analog transfer function is

$$H_a(s) = H(s) \Big| s \rightarrow \frac{s}{0.28\pi}$$

$$H_a(s) = \frac{1}{\left(\frac{s}{0.28\pi}\right)^2 + \sqrt{2}\left(\frac{s}{0.28\pi}\right) + 1}$$

$$\boxed{H_a(s) = \frac{0.5266}{s^2 + 1.035s + 0.5266}}$$

for impulse invariant method, solve the quadratic equation

$$s^2 + 1.035s + 0.5266 = -0.51 \pm j0.51$$

Thus

$$\frac{0.5266}{s^2+1.035+0.5266} = \frac{0.526}{\{s-(-0.51+j0.51)\}\{s-(-0.51-j0.51)\}}$$

By Partial fraction

$$= \frac{A}{s-(-0.5+j0.51)} + \frac{A^*}{(s-(-0.51-j0.51))}$$

By cross multiplication

$$0.5266 = A\{s-(-0.5-j0.51)\} + A^*(s-(-0.51+j0.51))$$

put $s = -0.51+j0.51$

$$\therefore A = \frac{0.526}{1.02j}$$

$$\boxed{A = -0.516j}$$

Since A^* is A conjugate so

$$\boxed{A^* = 0.516j}$$

S.S the value

$$Ha(s) = \frac{0.516j}{\{s-(-0.56-j0.516)\}} - \frac{0.56j}{s-(-0.5+j0.51)}$$

$$H(z) = \frac{0.516j}{1-e^{-0.5}e^{-j0.5}z^{-1}} - \frac{0.516j}{1-e^{-51}e^{j0.5}z^{-1}}$$

5. Design a chebyshev filter using (i) Impulse Invariene method (ii) Bilinear transformation method for the following specifications Dec 12

$$0.8 \le |H e^{j\omega}| \le 1 \qquad 0 \le \omega \le 0.2\pi$$

$$|H e^{j\omega}| \le 0.2 \qquad 0.6\pi \le \omega \le \pi.$$

Soln:

Given $\omega_s = 0.6\pi$

$\omega_p = 0.2\pi$

$$\frac{1}{\sqrt{1+\varepsilon^2}} = 0.8$$

$$\left(\sqrt{1+\varepsilon^2}\right)^2 = \frac{1}{(0.8)^2}$$

$$1+\varepsilon^2 = 1.5625$$

$$\boxed{\varepsilon^2 = 0.75}$$

$$\frac{1}{\sqrt{1+d^2}} = 0.2$$

$$1+d^2 = \frac{1}{(0.2)^2}$$

$$\boxed{d = 4.898}$$

(i) Bilinear Transformation

By prewraping the analog frequencies

$$\Omega_s = \frac{2}{T} \tan \frac{\omega_s}{2}$$

$$= 2 \tan\left(\frac{0.6\pi}{2}\right)$$

$$\Omega_s = 2.752.$$

$$\Omega_p = \frac{2}{T} \tan\left(\frac{\omega_p}{2}\right)$$

$$= 2 \tan\left(\frac{0.2\pi}{2}\right)$$

$$\boxed{\Omega_p = 0.6498}$$

To Find Order of the filter

$$N \geq \frac{\cosh^{-1}\left(d/\varepsilon\right)}{\cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)}$$

$$N \geq \frac{\cosh^{-1}\left(\frac{4.898}{6.75}\right)}{\cosh^{-1}\left(\frac{2.752}{0.6498}\right)}$$

$$N \geq 1.208$$

$$\boxed{N = 2}$$

To Find the values of a and b [major and Minor axis]

$$M = \varepsilon^{-1} + \sqrt{1+\varepsilon^{-2}}$$

$$M = \frac{1}{0.75} + \sqrt{1 + \frac{1}{(0.7)^2}}$$

$$M = 1.33 + 1.743$$

$$\boxed{M = 3.}$$

$$a = \Omega_p \left[\frac{\mu^{1/N} - \mu^{-1/N}}{2} \right]$$

$$= 0.6498 \left[\frac{3^{1/2} - 3^{-1/2}}{2} \right]$$

$$\boxed{a = 0.3752}$$

$$b = \Omega_p \left[\frac{\mu^{1/N} + \mu^{-1/N}}{2} \right]$$

$$= 0.6498 \left[\frac{3^{1/2} + 3^{-1/2}}{2} \right]$$

$$\boxed{b = 0.75}$$

To Find the poles of the Chebyshev filter

$$S_K = a \cos \varphi_K + j b \sin \varphi_K \quad K = 1, 2 \ldots N.$$

Where $\varphi_K = \frac{\pi}{2} + \left(\frac{2K-1}{} \right) \pi$

$$\varphi_1 = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$$

$$\varphi_2 = \frac{\pi}{2} + \frac{3\pi}{4} = \frac{5\pi}{4}.$$

P

then $s_1 = a\cos\phi + jb\sin\phi_1$

$= 0.3752 \cos\left(\frac{3\pi}{4}\right) + j\,0.75 \sin\left(3\pi/4\right)$

$s_1 = -0.2653 + j0.53.$

III^{ly} $s_2 = a\cos\phi_2 + j\sin\phi_2$

$= 0.3752 \cos\left(\frac{5\pi}{4}\right) + j\,0.75 \sin\left(5\pi/4\right)$

$s_2 = -0.2653 - j0.53.$

To Find $D(s)$

$D(s) = (s - s_1)(s - s_2)$

$= \{s - (-0.2653 + j\,0.53)\}\{s - \{-0.2653 - j\,0.53\}\}$

$= (s + 0.2653 - j\,0.53)(s + 0.2653 - j\,0.53)$

$= (s + 0.2653)^2 + (0.53)^2$

$$\boxed{D(s) = s^2 + 0.5306\,s + 0.3516.}$$

To Find $N(s)$

To find Numerator put $s = 0$ and $\frac{\epsilon}{\epsilon}$ the value by $\sqrt{1+\epsilon^2}$ for even.

$$N(s) = \frac{0.3516}{\sqrt{1+\epsilon^2}} = \frac{0.3516}{\sqrt{1+0.75^2}}$$

$N(s) = 0.28$

The transfer function $H(s) = N(s)/D(s)$

$$= \frac{0.28}{s^2 + 0.5306\,s + 0.3516}.$$

Apply Bilinear Transformation

substitute $s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)$ $T = 1\,s.$

$$H(s) = \frac{0.28}{\left\{\frac{2(1-z^{-1})}{(1+z^{-1})}\right\}^2 + \left\{0.5306 \times 2\left(\frac{1-z^{-1}}{1+z^{-1}}\right)\right\} + 0.3516.}$$

$$= \frac{0.28}{\frac{2^2(1-z^{-1})^2}{(1+z^{-1})^2} + 0.5306 \times 2\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 0.3516.}$$

$$H(z) = \frac{0.28\,(1+z^{-1})^2}{5.4128 - 7.298\,z^{-1} + 3.29\,z^{-2}}$$

(ii) In Impulse invarient Method.

The alalog frequencies are,

$$\Omega_s = \frac{\omega_s}{T} = 0.6\pi$$

$$\Omega_p = \frac{\omega_p}{T} = 0.2\pi$$

To Find order of the filter

$$N \geq \frac{\cosh^{-1}\left(\frac{d}{\varepsilon}\right)}{\cosh^{-1}\left(\frac{\Omega_s}{\Omega_p}\right)}$$

$$\geq \frac{\cosh^{-1}\left(\frac{4.898}{0.75}\right)}{\cos^{-1}\left(\frac{0.6\pi}{0.2\pi}\right)}$$

$$\geq 1.45$$

$$\boxed{N = 2}$$

To Find a and b

$$\mu = \varepsilon^{-1} + \sqrt{1+\varepsilon^2} = 0.75^{-1} + \sqrt{1+0.75^{-2}}$$

$$\boxed{\mu = 3}$$

$$a = \Omega_p \left[\frac{\mu^{1/N} - \mu^{-1/N}}{2}\right] = 0.2\pi \left[\frac{3^{1/2} - 3^{-1/2}}{2}\right]$$

$$= 0.3627$$

$$b = \Omega_p \left[\frac{\mu^{1/N} + \mu^{-1/N}}{2}\right] = 0.2\pi \left[\frac{3^{1/2} + 3^{-1/2}}{2}\right]$$

$$b = 0.7255$$

To Find poles.

$$s_k = a\cos\phi_k + j\sin\phi_k \qquad K = 1, 2 \ldots N.$$

$$\phi_k = \pi/2 + \frac{(2K+1)\pi}{2N} \qquad K = 1, 2.$$

$$\phi_1 = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$$

$$\phi_2 = \frac{\pi}{2} + \frac{3\pi}{4} = \frac{5\pi}{4}.$$

$S_1 = a\cos\phi_1 + j\,b\sin\phi_1$

$= 0.8627\cos\left(\frac{3\pi}{4}\right) + j\,0.7255\sin\left(\frac{3\pi}{4}\right)$

$S_1 = -0.2564 + j\,0.518.$

$S_2 = -0.2564 - j\,0.518.$

Numerator of $H(s) := (s - s_1)(s - s_2)\ldots$

$= (s - (-0.2564 + j0.518))(s - (-0.2564 - j0.518))$

$= [s - (-0.2564 + j0.518)][s - (-0.2564 - j0.518)]$

$= [\underbrace{s + 0.2564}_{a} - \underbrace{j\,0.5125}_{b}][\underbrace{s + 0.2564}_{a} + \underbrace{j\,0.518}_{b}]$

$= [s + 0.2564]^2 - [j0.5123]^2 \qquad [(a+b)(a-b) = a^2 - b^2]$

$= [s + 0.2564]^2 - j^2(0.5123)^2$

$= [s + 0.2564]^2 + (0.5125)^2$

$H(s) = [s + 0.2564]^2 + (0.5125)^2.$

For N_8 : S.S $s = 0$ & $\div$ by $\sqrt{1-\zeta^2} = 0.28.$

Numarator of H(s) for N = 2 (even).

S.S. $S = 0$ in the Dr of H(s) and $\pm\sqrt{1-\xi^2}$.

$$= \frac{(S+0.2564)^2 + (0.5125)^2}{\sqrt{1+(0.75)^2}} \Big/ S=0.$$

Nr of H(s) $= 0.2626$

$$\therefore H(s) = \frac{0.2626}{(S+0.2564)^2 + (0.5125)^2}$$

For Impulse invariant Method,

$$H(s) = \frac{0.2626}{S^2 + (0.2564)^2 + (2 \times 0.2564 \times S) + (0.5125)^2}$$

$$= \frac{0.2626}{S^2 + 0.0657 + 0.51256 + 0.2626.}$$

$$= \frac{0.2626}{S^2 + 0.5128\,S + 0.3283}$$

$$H(s) = \frac{0.2626}{[S-(-0.2564+j0.5124)][S-(-0.2564-j0.5124)]}$$

$$= \frac{A}{S-(-0.2564+j0.5124)} + \frac{A^*}{S-(-0.2564-j0.5124)}$$

P

$$A = \left[s - (-0.2564 + j\,0.5124)\right] \times \frac{0.2626}{(s-(-0.2564+j0.5124)(s-(-0.2564-j0.5124)}$$

$$= \frac{0.2626}{\left[s-(-0.2564-j\,0.5124)\right]} \Bigg|_{s=-0.2564+j\,0.5124}$$

$$= \frac{0.2626}{(-0.2564+j0.5124)-(-0.2564-j\,0.5124)}$$

$$= \frac{0.2626}{1.0248\,j}$$

$$= \frac{0.2626}{1.0248}(-j)$$

$$\boxed{A = -0.2562j}$$

$$\therefore \boxed{A^* = 0.2562j}$$

$$H(s) = \frac{-0.2562j}{s-(-0.2564+j0.5124} + \frac{0.2562j}{(s-(-0.2564-j\,0.5124)}$$

$$= -0.2562\left[\frac{1}{s-(-0.2564+j0.5124}\right] + 0.2562\left[\frac{1}{s-(-0.2564-j\,0.5124)}\right]$$

P.

W.K.T,

For Impulse invariant Method,

$$\frac{1}{S-P_i} \longrightarrow \frac{1}{1-e^{P_i T} z^{-1}}$$

$$\frac{1}{S-(-0.2564+j0.5124)} = \frac{1}{1-e^{(-0.2564+j0.5124)T} z^{-1}}$$

$$= \frac{1}{1-e^{(-0.2564+j0.5124)} z^{-1}}$$

$$= \frac{1}{1-e^{(-0.2564+j0.5124)} z^{-1}}$$

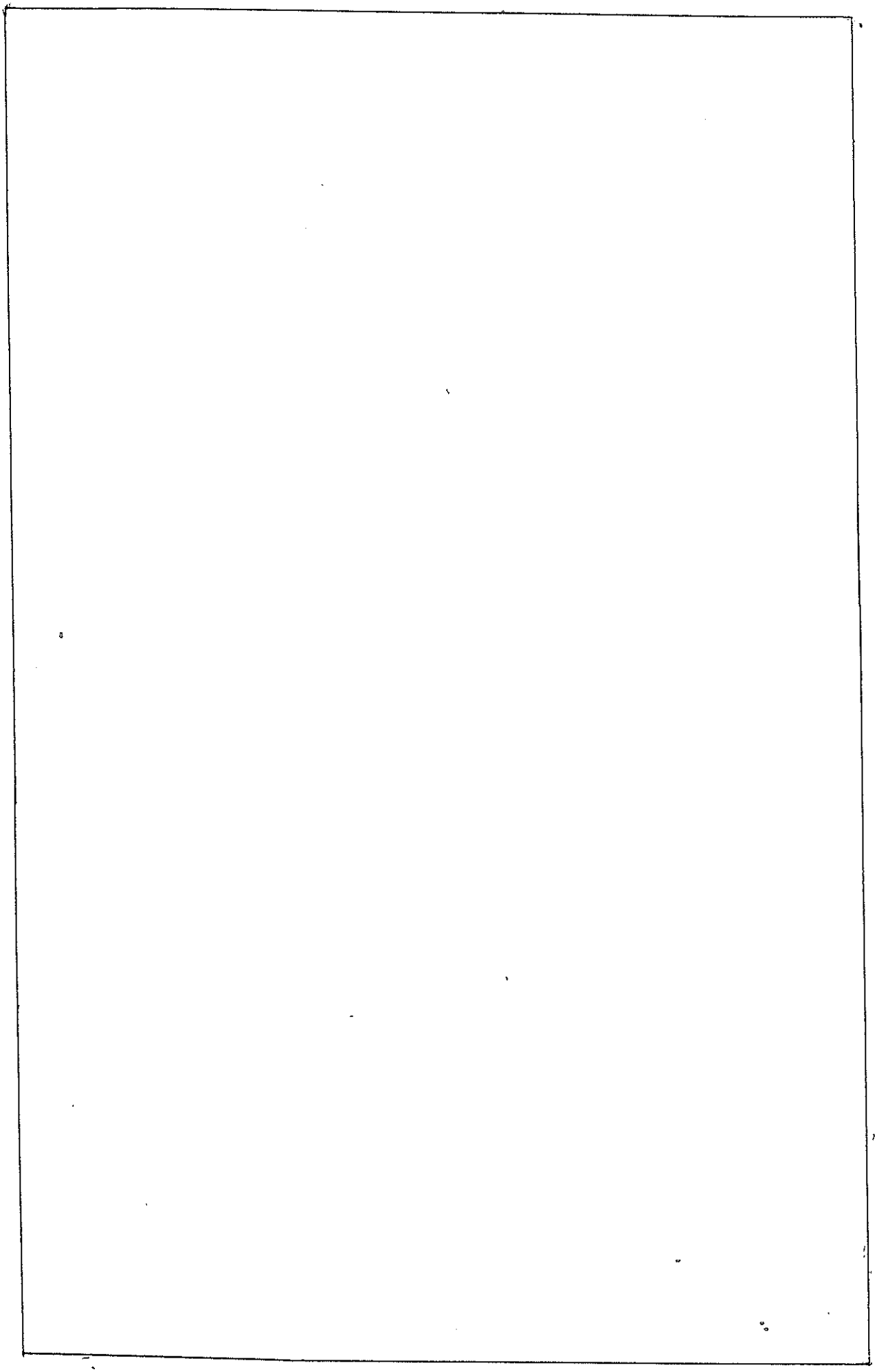
$$= \frac{1}{1-e^{-0.2564} e^{j0.5124} z^{-1}}$$

$$= \frac{1}{1-0.7738\ e^{j0.5124} z^{-1}}$$

$$\frac{1}{S-(-0.2564-j0.5124)} = \frac{1}{1-e^{(-0.2564-j0.512)}}$$

$$= \frac{1}{1-0.7738\ e^{-j0.5124}}$$

$$\therefore H(z) = 0.2562j\left[\frac{1}{1-0.7738\ e^{j0.5124} z^{-1}}\right] + 0.252j\left[\frac{1}{1-0.7738\ e^{-j0.512} z^{-1}}\right]$$



Pa

b Determine the system function H(z) of the lowest order chebyshev filter with the following specification

(a) 3 db ripple in passband $0 \le \omega \le 0.2\pi$

(b) 25 db attenuation in stop band $0.45\pi \le \omega \le \pi$

Soln

Using Bilinear transformation,

$\omega_p = 0.2\pi$; $\omega_s = 0.45\pi$, $\alpha_p = 3$ db

$\alpha_s = 25$ db

$$\Omega_p = \frac{2}{T} \tan \frac{\omega_p}{2} = \frac{2}{T} \tan(0.1\pi) = 0.65$$

$$\Omega_s = \frac{2}{T} \tan \frac{\omega_s}{2} = \frac{2}{T} \tan\left(\frac{0.45\pi}{2}\right) = 1.71$$

$$N \ge \frac{\cosh^{-1} \sqrt{\frac{10^{0.1\alpha_s} - 1}{10^{0.1\alpha_p} - 1}}}{\cosh^{-1} \frac{\Omega_s}{\Omega_p}} = \frac{\cosh^{-1} \sqrt{\frac{10^{2.5} - 1}{10^{0.3} - 1}}}{\cosh^{-1} \frac{\Omega_s}{\Omega_p}}$$

$$N = 3.$$

$$\varepsilon = \sqrt{10^{0.1\alpha_s} - 1} = 1$$

$$\mu = \varepsilon^{-1} + \sqrt{1 + \varepsilon^{-2}} = 2.414$$

Pa

$$a = \Omega_p \left[\frac{\mu^{1/N} - \mu^{-1/N}}{2} \right] = 0.65 \left[\frac{(2.414)^{1/3} - (2.414)^{-1/3}}{2} \right]$$

$$= 0.1935$$

$$b = \Omega_p \left[\frac{\mu^{1/N} + \mu^{-1/N}}{2} \right] = 0.65 \left[\frac{(2.414)^{1/3} + (2.414)^{-1/3}}{2} \right]$$

$$= 0.678$$

$$\phi_k = \frac{\pi}{2} + \frac{(2k-1)\pi}{2N} \qquad k = 1, 2, 3.$$

$$\phi_1 = \frac{\pi}{2} + \frac{\pi}{6} = 2\pi/3 \qquad \phi_2 = \frac{\pi}{2} + \frac{3\pi}{6} = \pi$$

$$\phi_3 = \frac{\pi}{2} + \frac{5\pi}{6} = \frac{8\pi}{6}$$

$$S_1 = 0.1935 \cos(\pi/3) + j\, 0.678 \sin\left(\frac{2\pi}{3}\right)$$

$$= -0.09675 + j\, 0.587.$$

$$S_2 = 0.1935 \cos(\pi) + j\, 0.678 \sin(\pi)$$

$$= -0.1935$$

$$S_3 = 0.1935 \cos(8\pi/6) + j\, 0.678 \sin\left(\frac{8\pi}{6}\right)$$

$$= -0.09675 - j\, 0.587.$$

The denominator polynomial HCs)

$$= (s + 0.1935)\left[(s + 0.09675)^2 + 0.587^2\right]$$

$$= (s + 0.1935)\left[s^2 + 0.19355 + 0.354\right]$$

The transfer of $H(s)$

The Numarator $\Rightarrow$ $s \cdot s$ $s = 0$. $[N = odd]$

$$= (0.1935)(0.354)$$
$$= 0.0685$$

The transfer function $H(s)$

$$H(s) = \frac{0.0685}{(s+0.1935)(s^2+0.1935s+0.354)}$$

$$H(z) = H(s)\Big|_{s = \frac{2(1-z^{-1})}{(1+z^{-1})}}$$

$$= \frac{0.0685}{\left(\frac{2(1-z^{-1})}{1+z^{-1}} + 0.1935\right)\left(\left\{\frac{2(1-z^{-1}}{1+z^{-1}}\right\}^2 + 0.1935\left(\frac{2(1-z^{-1}}{1+z^{-1}}\right) + 0.354\right.}$$

$$= \frac{0.0685\,[1+z^{-1}]^3}{\left(2(1-z^{-1}) + 0.1935(1+z^{-1})\right)\left[2(1-z^{-1})^2 + 0.1935)(2(1-z^{-1})^2\right]}$$

$$\boxed{H(s) = \frac{0.00687(1-z^{-1})^3}{(1-0.823z^{-1})(1-1.6z^{-1}+0.915z^{-2})}}$$

Design a third order Butterworth digital filter using impulse invariant technique. Assume Sampling period $T = 1$ sec.

Soln:

$N = 3$

$$H(s) = \frac{1}{(s+1)(s^2+s+1)}$$

$$= \frac{A}{s+1} + \frac{B}{s+0.5+j0.866} + \frac{C}{s+0.5-j0.866}$$

$$A = (s+1)\frac{1}{(s+1)(s^2+s+1)}\Big|_{s=-1} = \frac{1}{(-1)^2+1+1} = 1$$

$$B = (s+0.5+j0.866)\frac{1}{(s+1)(s+0.5+j0.866)(s+0.5-j0.8666)}\Big|_{s=0.5-j0.8666}$$

$$= \frac{1}{(-0.5-j0.866+1)(-j0.866-j0.866)}$$

$$= \frac{1}{(-j1.732(0.5-j0.866)} = \frac{1}{(-j0.866-j0.866)}$$

$$= \frac{-1.5+j0.866}{3} = -0.5+j0.288$$

$$C = B^* = -0.5-j0.288$$

Hence

$$H(s) = \frac{1}{s+1} + \frac{-0.5 + 0.288j}{s+0.5 + j0.866} + \frac{-0.5 - 0.288j}{s+0.5 - j0.866}$$

$$= \frac{1}{s-(-1)} + \frac{-0.5 + 0.288j}{s-(-0.5 - j0.866)} + \frac{-0.5 - 0.288j}{s-(-0.5 + j0.866)}$$

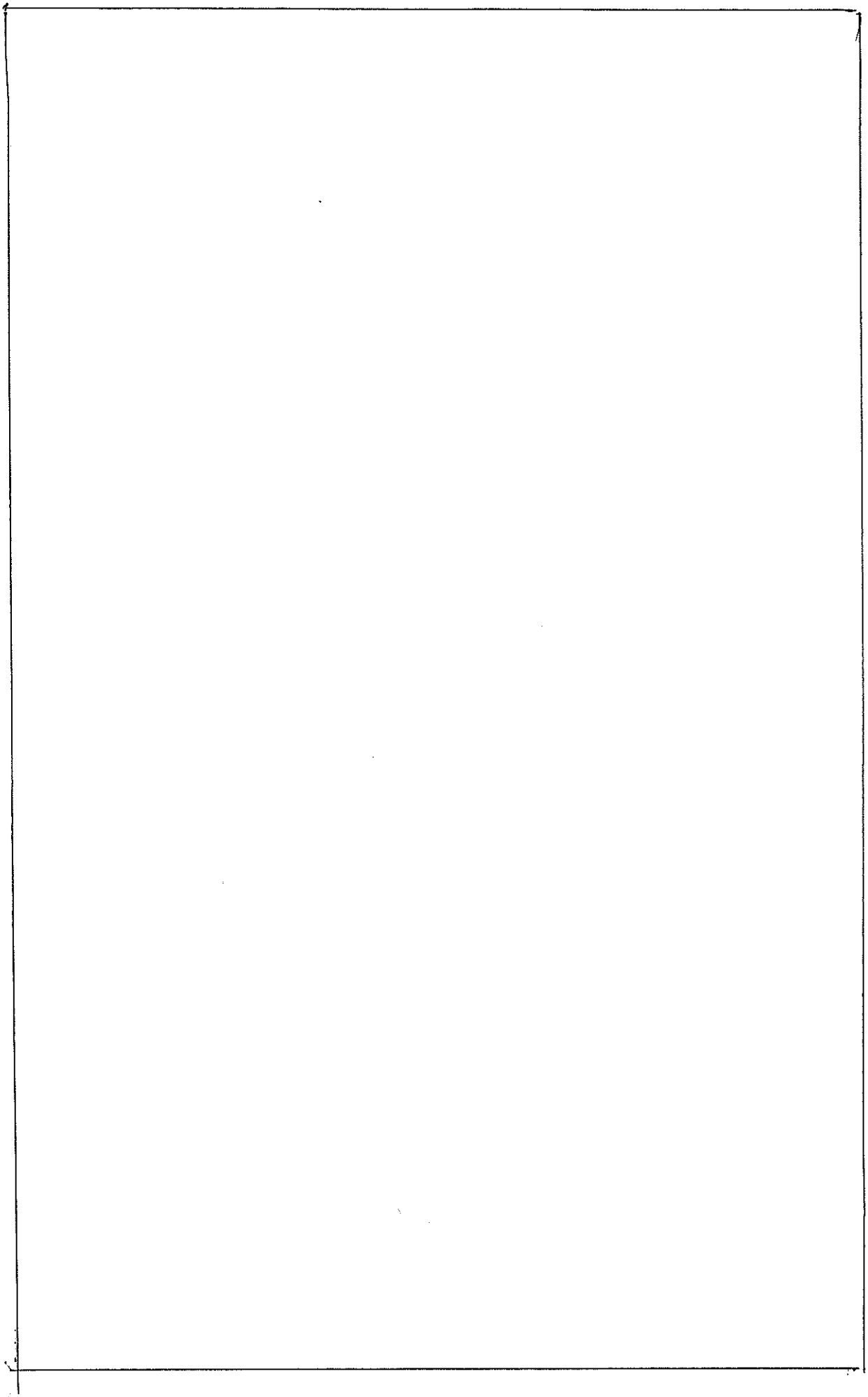
In Impulse invariant technique.

if $H(s) = \sum_{k=1}^{N} \frac{C_k}{s - P_k}$ then $H(z) = \sum_{k=1}^{N} \frac{C_k}{1 - e^{P_k T} z^{-1}}$

Therefore

$$H(z) = \frac{1}{1 - e^{1} z^{-1}} + \frac{-0.5 + j0.288}{1 - e^{-0.5} e^{-j0.866} z^{-1}} + \frac{-0.5 - j0.288}{1 - e^{-0.5} e^{+j0.866} z^{-1}}$$

$$H(z) = \frac{1}{1 - 0.368 z^{-1}} + \frac{-1 + 0.66 z^{-1}}{1 - 0.786 z^{-1} + 0.368 z^{-2}}$$



UNIT - III

FIR FILTER DESIGN.

Part - A (2 Marks)

1. What are the desirable characteristics of Window? Dec 2018.

* The central lobe of the frequency response of the window should contain most of the energy and should be narrow.

* The highest side lobe level of the frequency response should be small.

* The sidelobe of the frequency response should decrese in energy rapidly as ω tends to π.

2. What are the linear phase characteristics in FIR filter?

* The impulse response $h(n)$ of the system should have the symmetrical property.

$$h(n) = h(N-1-n)$$

* The phase function should be a linear function of ω, which in turn requires constant phase delay or group delay.

3. List out the advantages and disadvantages of FIR filters. June 13

Advt :

1. FIR filter have exact linear phase.
2. FIR filter always stable.
3. It can be realized in both recursive and non recursive structure.
4. Filters with any arbitary magnitude response can be tackled using FIR sequence.

Disadvt :

1. For the some filter specifications the order of the filter can be high as 5 to 10 times that of an IIR design.
2. Large storage requirements needed.
3. Powerful computational facilities required for the implementation.

4. State the Properties of FIR Filter. Dec 2013

1. FIR filter is always stable.
2. A realizable filter can always be obtained.
3. FIR filter has linear phase response.

5. What are called symmetric and antisymmetric FIR Filters.

$h(n) = h(N-1-n)$ symmetric response

$h(n) = -h(N-1-n)$ Antisymmetric Response

6. Why window function is used in FIR filter design?

*In FIR design by fourier series method, the infinite duration impulse response is truncated to finite duration impulse response.

* The abrupt truncation of impulse response introduces oscillations in the passband and stopband.

* The oscillations can be reduced through the use of less abrupt truncation of the fourier series.

* This can be achieved by multiplying the IIR by finite weighting sequence w(n) called a window.

7. Write the equation of Hamming Window function.

APR/MAY 2015
June 3'

$$W_H(n) = 0.54 + 0.46 \cos\left(\frac{2\pi n}{N-1}\right) ; \quad n = -\frac{N-1}{2} \text{ to } \frac{N-1}{2}$$

$$0 \qquad ; \quad 0.W.$$

$$W_H(n) = 0.54 - 0.46 \left(\cos\left(\frac{2\pi n}{N-1}\right) ; \quad n = 0 \text{ to } N-1\right.$$

$$0 \qquad 0.W.$$

8. Define Gibbs phenomenon.

* In FIR Filter design, In fourier series method, the infinite duration impulse response is truncated to finite duration impulse response.

* The abrupt truncation of Impulse response introduces oscillations in the passband and stopband. This effect is known as Gibbs phenomenon (or) Gibbs oscillations.

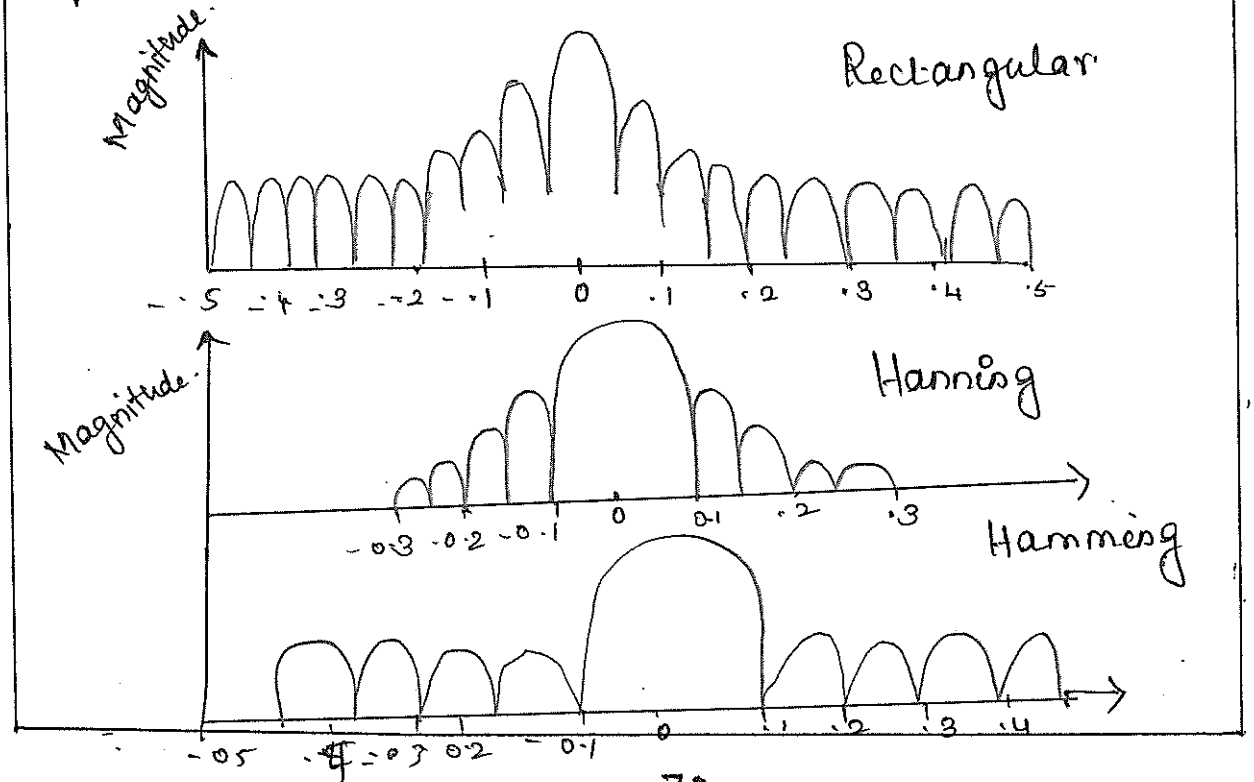
9. What are the Conditions to be satisfied for constant phase delay in Linear phase FIR filters?

The Conditions for Constant phase delay are

phase delay $\alpha = \frac{N-1}{2}$ (ie phase delay constant).

impulse response $h(n) = h(N-1-n)$.

10. Draw the shapes of any four window functions.



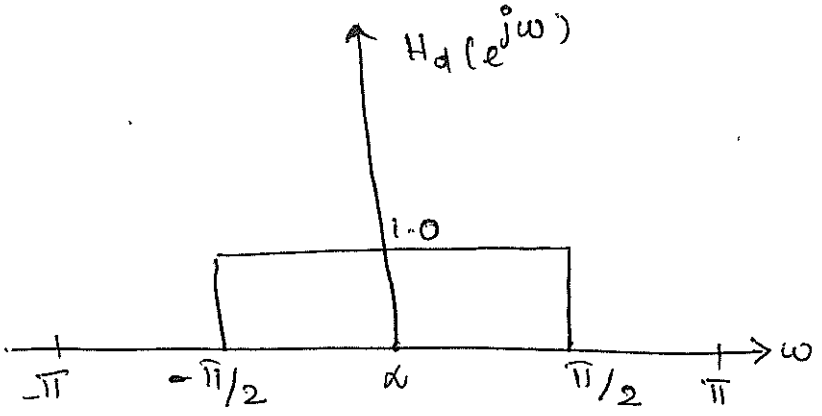
UNIT : III

1. Design an ideal lowpass filter with a frequency response $H_d(e^{j\omega}) = 1 \quad -\pi/2 \le \omega \le \pi/2$

$0 \quad \pi/2 \le \omega \le \pi$

Find the values of $h(n)$ for $N = 11$. Find $H(z)$ plot magnitude response.

Soln



$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi/2}^{\pi/2} e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi jn}\left[e^{j\omega n}\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{1}{\pi n(2j)}\left[e^{j\pi n/2} - e^{-j\pi n/2}\right]$$

$$= \frac{\sin \pi/2\, n}{\pi n} \quad -\infty \le n \le \infty$$

Truncating $h_d(n)$ to 11 samples, we have

$$h(n) = \frac{\sin \pi/2\, n}{\pi n} \quad \text{for } |n| \leq 5$$

$$= 0 \qquad \text{Otherwise.}$$

for $n = 0$ becomes indeterminate.

$$h(0) = \lim_{n \to 0} \frac{\sin \pi/2\, n}{\pi n} = \frac{1}{2} \lim_{n \to 0} \frac{\sin \pi/2\, n}{\pi/2}$$

apply L - Hospital rule, $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$

$$h(0) = \frac{1}{2}.$$

for $n = 1$ $\quad h(1) = h(-1) = \frac{\sin \pi/2}{\pi} = \frac{1}{\pi} = 0.3183.$

$n = 2$ $\quad h(2) = h(-2) = \frac{\sin 3\pi/2}{3\pi} = \frac{-1}{3\pi} = -0.106$

$n = 3$ $\quad h(3) = h(-3) = \frac{\sin 3\pi/2}{3\pi} = \frac{-1}{3\pi} = -0.106$

$n = 4$ $\quad h(4) = h(-4) = \frac{\sin 4\pi/2}{4\pi} = 0.$

$n = 5$ $\quad h(5) = h(-5) = \frac{\sin 5\pi/2}{5\pi} = \frac{1}{5\pi} = 0.06366$

The transfer function of the filter is given by

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} \left[h(n) \left(z^n + z^{-n} \right) \right]$$

$$= 0.5 + \sum_{n=1}^{5} h(n) \left(z^n + z^{-n} \right).$$

$$h(4) = h(-4) = \frac{\sin 4\pi/2}{4\pi} = 0.$$

$$h(5) = h(-5) = \frac{\sin 5\pi/2}{5\pi} = \frac{1}{5\pi} = 0.06366.$$

The transfer function of the filter is given by

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} \left[h(n)\left(z^{n} + z^{-n}\right) \right]$$

$$= 0.5 + \sum_{n=1}^{5} h(n)\left(z^{n} + z^{-n}\right)$$

$$= 0.5 + 0.3183\left(z^{+1} + z^{-1}\right) - 0.106\left(z^{3} + z^{-3}\right) + 0.06366\left(z^{5} + z^{-5}\right)$$

The transfer function of the realizable filter is

$$H'(z) = z^{-(N-1)/2} H(z)$$

$$= z^{-5}\left[0.5 + 0.3183\left(z + z^{-1}\right) - 0.106\left(z^{3} + z^{-3}\right) + 0.06366\left(z^{5} + z^{-5}\right)\right]$$

$$= 0.06366 - 0.106z^{-2} + 0.3183z^{-4} + 0.5z^{-5} + 0.3183z^{-6} - 0.106z^{-8} + 0.06366z^{-10}.$$

$h(0) = h(10) = 0.06366$ $\quad h(1) = h(9) = 0$ $\quad h(2) = h(8) = -0.106$

$h(3) = h(7) = 0$ $\quad h(4) = h(6) = 0.3183$ $\quad h(5) = 0.5$

The frequency response is given by

$$H(e^{j\omega}) = \sum_{n=0}^{5} a(n)\cos\omega n \quad \text{where,}$$

$$a(0) = h\left[\frac{N-1}{2}\right] = h(5) = 0.5.$$

$$a(n) = 2h\left(\frac{N-1}{2} - n\right)$$

$a(1) = 2h(5-1) = 2h(4) = 0.6366$

$a(2) = 2h(5-2) = 2h(3) = 0$

$a(3) = 2h(5-3) = 2h(2) = -0.212$

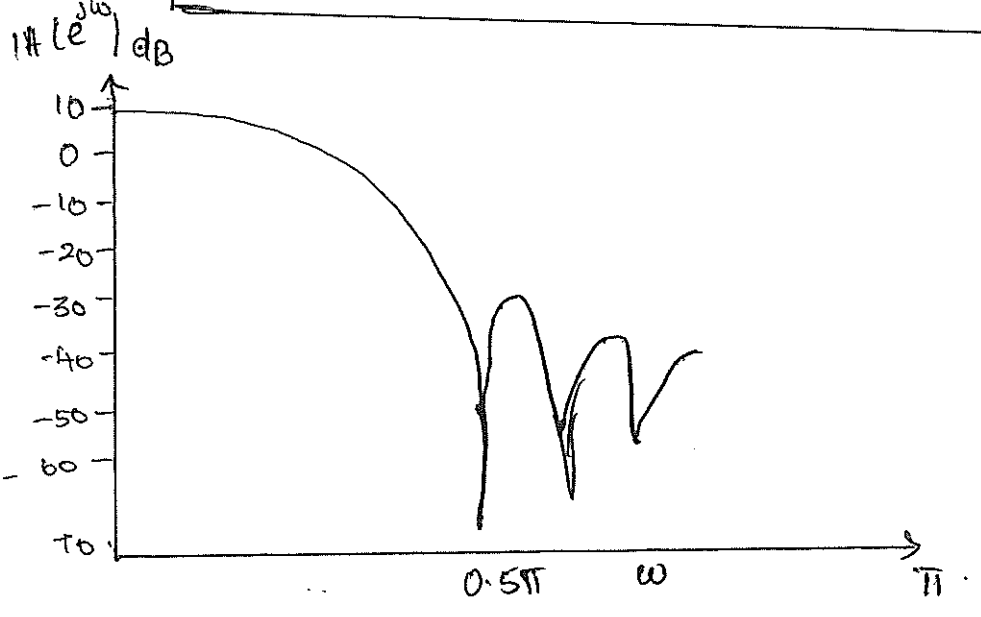
$a(4) = 2h(5-4) = 2h(1) = 0.$

$a(5) = 2h(5-5) = 2h(0) = 0.127.$

$$H(e^{j\omega}) = 0.5 + 0.6366 \cos\omega - 0.212 \cos 3\omega + 0.127 \cos 5\omega.$$

ω in degrees	0	10	20	30	40	50	60	70	80
$\lvert H(e^{j\omega})\rvert_{dB}$	0.4	0.21	-0.26	-0.514	-0.21	0.42	0.77	0.21	-1.79

90	100	110	120	130	140	150	160
-6	-14.56	-31.89	-20.6	-26	-32	24.4	-30.55



2. Using Rectangular Window technique design a low pass filter with passband gain of unity Cut-off frequency of 1000 Hz and working at a sampling frequency of 5 KHz. The length of the impulse response should be 7.

Apr/may 2015
Dec 2013.

Soln :

Given $f_c = 1000$ Hz. ; $F_s = 5$ KHz.

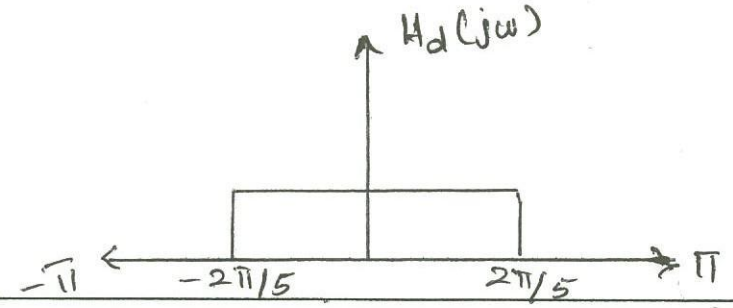
$N = 7$.

$$\omega_c = \frac{2\pi f_c}{F_s} = \frac{2\pi \times 1000}{5 \times 10^3} = \frac{2\pi}{5} = 1.25 \text{ rad/sec}$$

The desired frequency response for LPF

$$H_d(e^{j\omega}) = \begin{cases} 1 & \text{for } -\omega_c \le \omega \le \omega_c \\ 0 & \text{for O.W} \end{cases}$$

$$H_d(e^{j\omega}) = \begin{cases} 1 & \text{for } \frac{-2\pi}{5} \le \omega \le \frac{2\pi}{5} \\ 0 & \text{for O.W.} \end{cases}$$



To find $h_d(n)$

$$h_d(n) = \frac{\sin \omega_c n}{\pi n} = \frac{\sin \frac{2\pi}{5} n}{\pi n}.$$

The filter coefficient for LPF is

$$h_d(0) = \frac{\omega_c}{\pi} = \frac{2\pi}{5} \times \frac{1}{\pi} = 0.4$$

$$h_d(1) = h_d(-1) = \frac{\sin 2\pi/5}{\pi} = 0.302$$

$$h_d(2) = h_d(-2) = \frac{\sin 4\pi/5}{2\pi} = 0.093$$

$$h_d(3) = h_d(-3) = \frac{\sin 6\pi/5}{2\pi} = -0.062.$$

To Find Window Sequence

The rectangular window function

$$W_R(n) = \begin{cases} 1 & \text{for } -\left(\frac{N-1}{2}\right) \le n \le \left(\frac{N-1}{2}\right) \\ 0 & \text{for } \text{O.W.} \end{cases}$$

$$W_R(n) = \begin{cases} 1 & -3 \le n \le 3 \\ 0 & \text{o.w.} \end{cases}$$

$W_R(0) = 0$

$W_R(1) = W_R(-1) = 1$

$W_R(2) = W_R(-2) = 1$

$W_R(3) = W_R(-3) = 1$

To find $H_d(n)$

$$h(n) = h_d(n)\, w_R(n)$$

$h(0) = 0.4$

$h(1) = h(-1) = 0.302$

$h(2) = h(-2) = 0.093$

$h(3) = h(-3) = -0.062.$

To Find $H(z)$

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)\left(z^{n} + z^{-n}\right)$$

$$= 0.4 + h(1)\left[z^{1} + z^{-1}\right] + h(2)\left(z^{2} + z^{-2}\right) + h(3)\left(z^{3} + z^{-3}\right)$$

$$H(z) = 0.4 + 0.302\left[z + z^{-1}\right] + 0.093\left[z^{2} + z^{-2}\right] - 0.062\left[z^{3} + z^{-3}\right]$$

---

3. Design a BPF to pass a frequency rate 1 – 2 rad/sec. using hanning window $N = 5$.

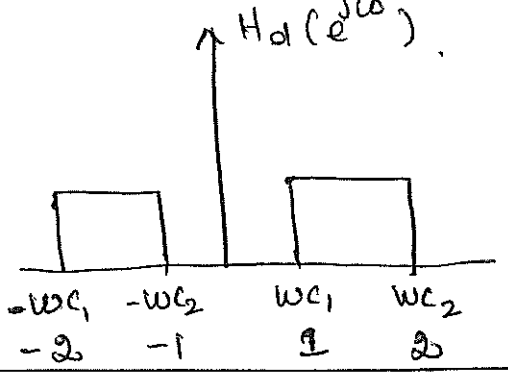
June 13.

Given.

$\omega c_1 = 1$ rad/sec.

$\omega c_2 = 2$ rad/sec.

$N = 5$.

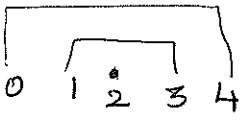


P

The desired frequency response for linear phase filter

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\alpha} & ; \quad \omega c_2 \le |\omega| \le \omega c_1 \\ 0 & \text{o.w.} \end{cases}$$

$$\alpha = \frac{N-1}{2} = \frac{5-1}{2} = 2$$



To Find $h_d(n)$

$$h_d(n) = \frac{\sin \omega c_2 (n-\alpha) - \sin \omega c_1 (n-\alpha)}{(n-\alpha)\pi}$$

$$h_d(n) = \frac{\sin \omega c_2 (n-2) - \sin \omega c_1 (n-2)}{\pi (n-2)}$$

$$= \frac{\sin 2(n-2) - \sin (n-2)}{\pi (n-2)}$$

For symmetric conditions $h(n) = h(N-1-n)$

$h(0) = h(4)$ $\quad h(2) = \alpha.$

$h(1) = h(3)$

$$h_d(0) = \frac{\sin(-4) - \sin(-2)}{-2\pi} = -0.265 = h(4)$$

$$h_d(1) = \frac{\sin(-2) - \sin(-1)}{-\pi} = 0.0201 = h_d(3)$$

$$h_d(2) = \frac{0}{0}$$

$$\therefore h_d(2) = \frac{\omega c_2 - \omega c_1}{\pi} = \frac{2-1}{\pi} = 1/\pi.$$

$$h_d(2) = 0.318.$$

To Find window sequence,

The <u>Causal</u> window function

$$W_{Hn}(n) = \begin{cases} 0.5 - 0.5\cos\left(\frac{2\pi n}{N-1}\right) & 0 \le n \le N-1 \\ 0 & \text{o.w.} \end{cases}$$

$$W_{Hn}(n) = \begin{cases} 0.5 - 0.5\cos\left(\frac{2\pi n}{4}\right) & 0 \le n \le 4. \end{cases}$$

$$W_{Hn}(0) = W_{Hn}(4) = 0.5 - 0.5 = 0.$$

$$W_{Hn}(1) = W_{Hn}(3) = 0.5 - 0.5(0) = 0.5$$

$$W_{Hn}(2) = 0.5 - 0.5(-1) = \underline{1}.$$

To find $h(n)$

$$h(n) = h_d(n)\, W_{Hn}(n).$$

$$h(0) = h_d(0) \times W_{Hn}(0) = -0.265 \times 0 = 0. = h(4)$$

$$h(1) = h_d(1) \times W_H(1) = 0.0201 \times 0.5 = 0.0105 = h(3)$$

$$h(2) = h_d(2) \times W_H(2) = 0.318 \times 1 = 0.318.$$

To find $H(z)$

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)\left[z^{n} + z^{-n}\right]$$

$$= h(0) + \sum_{n=1}^{2} h(n)\left[z^{n} + z^{-n}\right]$$

$$= h(0) + h(1)\left[z + z^{-1}\right] + h(2)\left[z^{2} + z^{-2}\right]$$

$$H(z) = 0.0105\left[z + z^{-1}\right] + 0.318\left[z^{2} + z^{-2}\right]$$

To find frequency response $z = e^{j\omega}$

$$H(e^{j\omega}) = 0.0105\left[e^{j\omega} + e^{-j\omega}\right] + 0.318\left[e^{j2\omega} + e^{-j2\omega}\right]$$

$$= 0.0105 \times 2\cos\omega + 0.318 \times 2\cos 2\omega$$

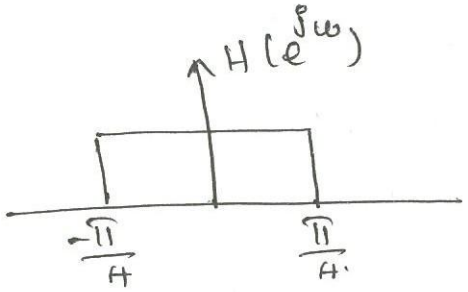
$$\therefore \boxed{H(e^{j\omega}) = 0.021\cos\omega + 0.636\cos 2\omega}$$

4. Design an ideal filter with the frequency response $H(e^{j\omega}) = \begin{cases} e^{-j3\omega} & \text{for } \frac{-\pi}{4} \le \omega \le \frac{\pi}{4} \\ 0 & \text{for } 0 < \omega < \frac{-\pi}{4} \text{ and } \frac{\pi}{4} < \omega < \pi \end{cases}$

Determine $h(n)$ for $N = 7$ use Hamming Window.

Apr/May 2015
Dec 12.

Soln:



The given filter is LPF, $\alpha = \frac{N-1}{2}$

To find $h(n)$

$$h(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{j\omega}) \cdot e^{j\omega n} \, d\omega.$$

$$= \frac{1}{2\pi} \int_{-\pi/4}^{\pi/4} e^{-j3\omega} \cdot e^{j\omega n} \, d\omega.$$

$$= \frac{1}{2\pi} \int_{-\pi/4}^{\pi/4} e^{-j\omega(3-n)} \cdot d\omega.$$

$$= \frac{1}{2\pi} \int_{-\pi/4}^{\pi/4} e^{-j\omega(3-n)} \cdot d\omega.$$

$$= \frac{1}{2\pi} \left[\frac{e^{-j\omega(3-n)}}{-j(3-n)} \right]_{-\pi/4}^{\pi/4}.$$

$$= \frac{-1}{2\pi j(3-n)} \left[e^{-j\pi/4(3-n)} - e^{j\pi/4(3-n)} \right]$$

$$= \frac{1}{2\pi j(3-n)} \left[e^{j\pi/4(3-n)} - e^{-j\frac{\pi}{4}(3-n)} \right]$$

$$= \frac{1}{\pi(3-n)} \sin\frac{\pi}{4}(3-n)$$

$$h_d(n) = \frac{\sin \pi/4(n-3)}{\pi(n-3)}$$

The desired filter co-efficient for symmetric Condition $h(n) = h(N-1-n)$ with $N=7$ is given as,

$$h_d(0) = h_d(6) = \frac{\sin(-3\pi/4)}{-3\pi} = 0.075$$

$$h_d(2) = h_d(4) = \frac{\sin -\pi/4}{\pi} = 0.225$$

$$h_d(1) = h_d(5) = \frac{\sin(-\pi/2)}{-2\pi} = 0.159$$

$$h(3) = \frac{\omega_c}{\pi} = \frac{\pi}{4} = \frac{1}{4} = 0.25$$

To Find the Window Sequence

The causal hamming window sequence is

$$W_{Hm} = 0.54 - 0.46 \cos\left(\frac{2\pi n}{N-1}\right); \quad 0 \le n \le N-1$$

$$W_{Hm}(n) = 0.54 - 0.46 \cos\left(\frac{2\pi n}{6}\right) \quad 0 \le n \le 6.$$

$W_{Hm}(0) = 0.54 - 0.46 = 0.08 = W_{Hm}(6)$.

$W_{Hm}(1) = 0.54 - 0.46 \cos\left(\frac{\pi}{3}\right) = 0.31 = W_{Hm}(5)$

$W_{Hm}(2) = 0.54 - 0.46 \cos\left(\frac{2\pi}{3}\right) = 0.77 = W_{Hm}(4)$

$W_{Hm}(3) = 0.54 - 0.46 \cos\pi = 1.$

To find $h(n)$

$$h(n) = h_d(n) \cdot W_{Hm}(n).$$

$h(6) = h(0) = h_d(0) \cdot W_H(0) = 0.075 \times 0.08 = 0.006.$

$h(5) = h(1) = h_d(1) \cdot W_H(1) = 0.049.$

$h(2) = h(4) = h_d(2)\ W_H(2) = 0.173.$

$h(3) = h_d(3)\ W_{Hm}(2) = 0.25.$

To find $H(z)$

$$H(z) = h(0) + \sum_{n=1}^{\frac{N-1}{2}} h(n)\left[z^n + z^{-n}\right]$$

$$H(z) = h(0) + \sum_{n=1}^{3} h(n)\left[z^n + z^{-n}\right].$$

$$H(z) = h(0) + h(1)(z + z^{-1}) + h(2)(z^2 + z^{-2}) + h(3)(z^3 + z^{-3}).$$

$$H(z) = 0.006 + 0.049(z + z^{-1}) + 0.173(z^2 + z^{-2}) + 0.25(z^3 + z^{-3}.$$

5. Derive the frequency response of a linear phase FIR filter for symmetrical Impulse response when N = odd.

Proof :-

The Fourier Transform of h(n) is

$$H(e^{jw}) = \sum_{n=0}^{N-1} h(n)\, e^{-jwn} \qquad \text{——— (1)}$$

When N is odd, Number, the symmetrical impulse response will have the centre of symmetry at $n = \frac{N-1}{2}$. Hence $H(e^{jw})$ is,

$$H(e^{jw}) = \sum_{n=0}^{\frac{N-3}{2}} h(n)\, e^{-jwn} + h\left(\frac{N-1}{2}\right) e^{-jw\left(\frac{N-1}{2}\right)} + \sum_{n=\frac{N+1}{2}}^{N-1} h(n).\, e^{-jwn}.$$

Let us modify the limits of summation

Put $n = N-1-m$;

$m = N-1-n$.

if $n = \frac{N+1}{2}$

$m = N-1-\left(\frac{N+1}{2}\right)$

$\boxed{m = \frac{N-3}{2}.}$

if $n = N-1$

$m = N-1-(N-1)$

$\boxed{m = 0.}$

(2) $\Rightarrow H(e^{j\omega}) = \sum_{n=0}^{\frac{N-3}{2}} h(n) e^{-j\omega n} + h\left(\frac{N-1}{2}\right) e^{-j\left(\frac{N-1}{2}\right)} + \sum_{n=0}^{\frac{N-3}{2}} h(N-1-m)\, e^{-j\omega(N-1-m)}$

Let $m = n$

$$H(e^{j\omega}) = \sum_{n=0}^{N-3/2} h(n) e^{-j\omega n} + h\left(\frac{N-1}{2}\right) e^{-j\left(\frac{N-1}{2}\right)} + \sum_{n=0}^{\frac{N-3}{2}} h(N-1-n)\, e^{-j\omega(N-1-n)}$$

For symmetrical $h(n) = h(N-1-n)$

$$H(e^{j\omega}) = \sum_{n=0}^{\frac{N-3}{2}} h(n) e^{-j\omega n} + h\left(\frac{N-1}{2}\right) e^{-j\left(\frac{N-1}{2}\right)} + \sum_{n=0}^{\frac{N-3}{2}} h(n)\, e^{-j(\omega(N-1)-n)}$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \left[\sum_{n=0}^{\frac{N-3}{2}} h(n) e^{-j\omega n} e^{j\omega\left(\frac{N-1}{2}\right)} + h\left(\frac{N-1}{2}\right) + \sum_{n=0}^{\frac{N-3}{2}} h(n)\, e^{-j(N-1-n)} e^{j\omega\left(\frac{N-1}{2}\right)} \right]$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \left[\sum_{n=0}^{\frac{N-3}{2}} h(n) e^{j\omega\left(\frac{N-1}{2}-n\right)} + h\left(\frac{N-1}{2}\right) + \sum_{n=0}^{\frac{N-3}{2}} h(n) e^{-j\omega\left(\frac{N-1}{2}-n\right)} \right]$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \left\{ \sum_{n=0}^{\frac{N-3}{2}} h(n) \left[e^{j\omega\left(\frac{N-1}{2}-n\right)} + e^{-j\omega\left(\frac{N-1}{2}-n\right)} \right] + h\left(\frac{N-1}{2}\right) \right\}$$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \left\{ \sum_{n=0}^{\frac{N-3}{2}} h(n)\, 2\cos\omega\left(\frac{N-1}{2}-n\right) + h\left(\frac{N-1}{2}\right) \right\}$$

Let $P = \frac{N-1}{2} - n$

$n = \frac{N-1}{2} - P$

if $n = 0$ $\quad \boxed{P = \frac{N-1}{2}}$

if $n = \frac{N-3}{2}$ $\quad P = \frac{N-1}{2} - \left(\frac{N-3}{2}\right)$

$\boxed{P = 1}$

$$= e^{-j\omega\left(\frac{N-1}{2}\right)} \left\{ \sum_{P=1}^{\frac{N-1}{2}} 2h\left(\frac{N-1}{2} - P\right) \cos\omega P + h\left(\frac{N-1}{2}\right) \right\}$$

Let $a(0) = h\left(\frac{N-1}{2}\right)$

$a(n) = 2h\left(\frac{N-1}{2} - P\right).$

$$H(e^{j\omega}) = e^{-j\omega\alpha} \left\{ \sum_{P=1}^{\frac{N-1}{2}} a(n) \cos\omega_P + a(0) \right\}$$

Let $P = n$

$$H(e^{j\omega}) = e^{-j\omega\alpha} \left\{ a(0) + \sum_{n=1}^{\frac{N-1}{2}} a(n) \cos\omega n \right\}$$

$$H(e^{j\omega}) = e^{-j\omega\alpha} \sum_{n=0}^{N-1/2} a(n) \cos\omega n.$$

6. Using frequency sampling method design a band pass filter with a following specifications. Sampling frequency $f = 8000$ Hz. Cut off frequency $f_{c_1} = 1000$ Hz $f_{c_2} = 3000$ Hz. for $N = 7$.

June 13. Dec 12

Soln:

$$\omega_{c_1} = \frac{2\pi f_{c_1}}{F} = \frac{2\pi \times 1000}{8000} = \frac{\pi}{4}.$$

$$\omega_{c_2} = \frac{2\pi f_{c_2}}{F} = \frac{2\pi \times 3000}{8000} = \frac{3\pi}{4}.$$

The desired frequency response

$$H(\omega) = \begin{cases} e^{-j\omega\alpha} & \text{for } -\omega_{c_1} \le \omega \le \omega_{c_2} \\ 0 & \text{for } 0.w. \end{cases}$$

where $\alpha = \frac{N-1}{2} = \frac{7-1}{2} = 3$.

Here we are going to sample the freq. in five values only

$$H(\omega) = \begin{cases} e^{-j3\omega} & \text{for } \omega_{c_1} \le \omega \le \omega_{c_2} \\ 0 & \text{for } 0.w. \end{cases}$$

To find $H(k)$

$$H(k) = H(\omega)\Big|_{\omega = \frac{2\pi k}{N}} = \frac{2\pi k}{7}.$$

$$H(k) = \begin{cases} e^{\frac{-j6\pi k}{7}} & \text{for } \frac{\pi}{4} \le \frac{2\pi k}{7} \le \frac{3\pi}{4} \\ 0 & \text{O.W} \end{cases}$$

$$H(k) = \begin{cases} e^{\frac{-j6\pi k}{7}} & ; \frac{7}{8} \le k \le \frac{21}{8} \\ 0 & \text{O.W.} \end{cases}$$

$$H(k) = \begin{cases} e^{\frac{-j6\pi k}{7}} & 0.875 \le k \le 2.625 \\ 0 & \text{O.W.} \end{cases}$$

$$H(k) = \begin{cases} e^{-j6\pi k/7} & 1 \le k \le 2 \\ 0 & \text{O.W.} \end{cases}$$

To Find $h(n)$

$$h(n) = \frac{1}{N} \left\{ H(0) + 2 \sum_{k=1}^{\frac{N-1}{2}} Re \left[H(k)\, e^{\frac{j2\pi kn}{N}} \right] \right\}$$

$$= \frac{1}{7} \left\{ H(0) + 2 \sum_{k=1}^{3} Re \left[e^{\frac{-j6\pi k}{7}} e^{\frac{j2\pi kn}{N}} \right] \right\}$$

$$= \frac{1}{7} \left\{ \left[2 \sum_{k=1}^{2} Re \left[e^{\frac{j2\pi k(n-8)}{7}} \right] \right] \right\}$$

$$= \frac{1}{7} \left[2 \sum_{k=1}^{2} \cos \left(\frac{2\pi k(n-8)}{7} \right) \right]$$

$$h(n) = \frac{1}{7}\left[2\cos\frac{2\pi(n-3)}{7} + 2\cos\frac{4\pi(n-3)}{7}\right]$$

To find Filter Co-efficient-

$h(0) = h(6) = -0.079$

$h(1) = h(5) = -0.32$

$h(2) = h(4) = 0.114.$

$h(3) = 0.571.$

To Find $H(z)$

$$H(z) = \sum_{n=0}^{N-1} h(n)\, z^{-n} = \sum_{n=0}^{6} h(n)\, z^{-1}$$

$$= h(0) + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3} + h(4)z^{-4} + h(5)z^{-5} + h(6)z^{-6}.$$

$$H(z) = -0.079 - 0.32z^{-1} + 0.114z^{-2} + 0.571z^{-3} - 0.114z^{-4} - 0.32z^{-5} - 0.079z^{-6}.$$

$$H(z) = -0.079\left[1+z^{-6}\right] + 0.32\left[z^{-1}+z^{-5}\right] + 0.114\left[z^{-2}+z^{-4}\right] + 0.571\,z^{-3}.$$

$$H(z)z^{3} = -0.079\left[z^{3}+z^{-3}\right] + 0.32\left[z^{2}+z^{-2}\right] + 0.114\left[z+z^{-1}\right] + 0.571.$$

$$H(z) = z^{-3}\left[-0.079\left[z^{3}+z^{-3}\right] + 0.32\left[z^{2}+z^{-2}\right] + 0.114\left[z+z^{-1}\right] + 0.571\right].$$

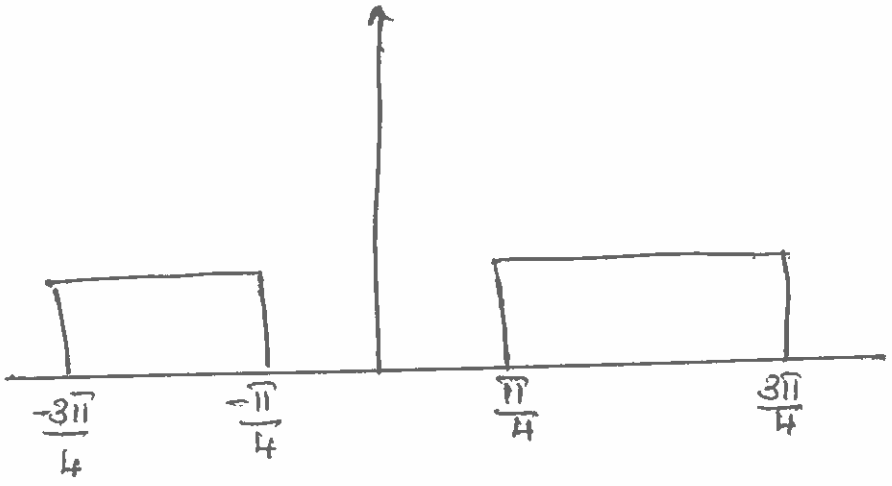
Design an ideal BPF with a frequency response

$$H_d(e^{j\omega}) = 1 \quad \text{for } \frac{\pi}{4} \le |\omega| \le \frac{3\pi}{4}$$

$$= 0 \quad \text{O.W.}$$

Find the values of $h(n)$ for $N = 11$ and plot the frequency response

Solution



$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}$$

$$= \frac{1}{2\pi} \left[\int_{-\frac{3\pi}{4}}^{-\pi/4} e^{j\omega n}\, d\omega + \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} e^{j\omega n}\, d\omega \right]$$

$$= \frac{1}{2\pi j n} \left[e^{-j\pi n/4} - e^{-j3\pi n/4} + e^{j3\pi n/4} - e^{j\pi n/4} \right]$$

$$= \frac{1}{\pi n} \left[\sin\frac{3\pi}{4} n - \sin\frac{\pi}{4} n \right] \quad -\infty < n < \infty$$

Truncating $h_d(n)$ to 11 samples we have

$$h(n) = h_d(n) \quad \text{for } |n| \le 5$$

$$0 \quad \text{otherwise.}$$

The filter co-efficients are symmetrical about $n=0$ satisfying the conditions $h(n) = h(-n)$.

for $n=0$

$$h(0) = \frac{1}{2\pi}\left[\int_{-\frac{3\pi}{4}}^{\frac{-\pi}{4}} d\omega + \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} d\omega\right]$$

$$= \frac{1}{2\pi}\left[-\frac{\pi}{4} + \frac{3\pi}{4} + \frac{3\pi}{4} - \frac{\pi}{4}\right] = \frac{1}{2} = 0.5$$

$h(0) = h(10) = 0.045$ $\quad h(1) = h(9) = 0$ $\quad h(2) = h(8) = -0.075$

$h(3) = h(7) = -0.159$ $\quad h(4) = h(6) = -0.225$

$h(5) = 0.75$

$$H(e^{j\omega}) = \sum_{n=0}^{\frac{N-1}{2}} a_n \cos\omega n)$$

$$a(0) = h\left(\frac{N-1}{2}\right) = h(5) = 0.75$$

$$a(n) = 2h\left(\frac{N-1}{2} - n\right)$$

$a(1) = 2h(5-1) = 2h(4) = -0.45$

$a(2) = 2h(5-2) = 2h(3) = -0.318$

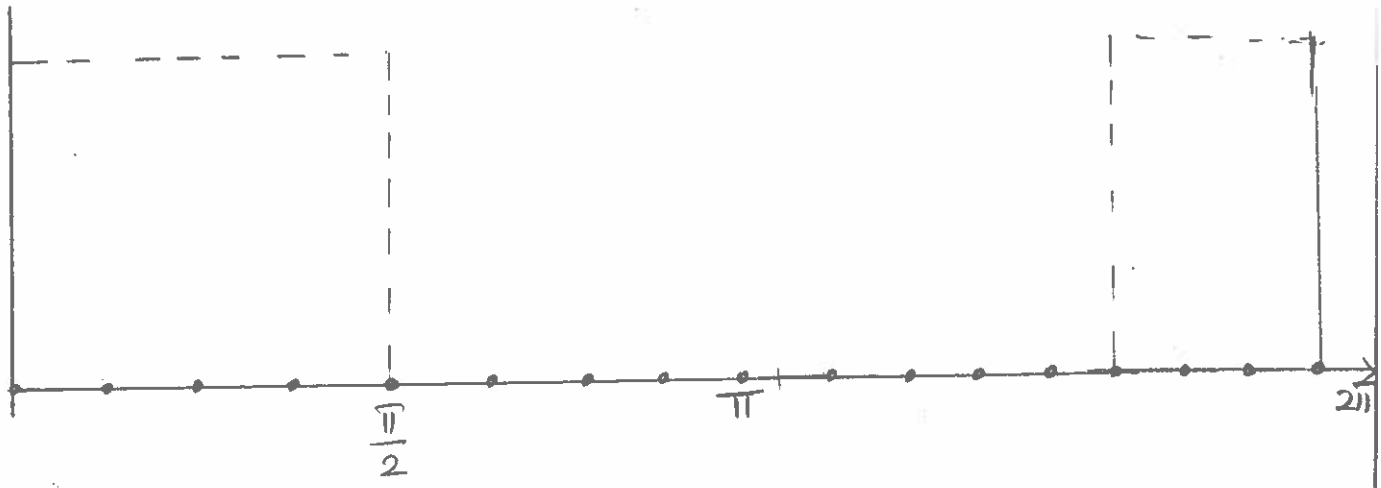
$a(3) = -0.15$

$a(4) = 0$ $\quad a(5) = 0.09$.

$$H(e^{j\omega}) = 0.75 - 0.45\cos\omega - 0.318\cos 2\omega - 0.15\cos 3\omega + 0.09\cos 5\omega.$$

Design a Linear phase filter with a Cut off frequency of $\frac{\pi}{2}$ rad/sec taken N=17 using frequency sampling technique.

Solution: $\omega_c = \frac{\pi}{2}$ $N = 17$.



$$H(K) = \begin{cases} e^{-j16\pi K/17} & \text{for } K = 0,1,2,3,4. \\ 0 & \text{for } 5 \le K \le 12 \\ e^{-j16(K-17)/17} & \text{for } 13 \le K \le 17. \end{cases}$$

$$h(n) = \frac{1}{N}\left[H(0) + 2\sum_{K=1}^{\frac{N-1}{2}} Re\left(H(K)e^{j2\pi Kn/N}\right)\right]$$

$$= \frac{1}{17}\left[1 + 2\sum_{K=1}^{8} Re\left(H(K)e^{j2\pi Kn/N}\right)\right]$$

$$= \frac{1}{17}\left[1 + 2 \sum_{K=1}^{8} Re\left(e^{-j16\pi K/17} \; e^{j2\pi Kn/17}\right)\right]$$

$$= \frac{1}{17}\left[1 + 2 \cos\frac{2\pi}{17}(8-n)\right]$$

$h(0) = h(16) = -0.056$

$h(1) = h(15) = -0.041$

$h(2) = h(14) = -0.0136$

$h(3) = h(13) = 0.026$

$h(4) = h(12) = 0.0698$

$h(5) = h(11) = 0.111$

$h(6) = h(10) = 0.145$

$h(7) = h(9) = 0.168$

$h(8) = 0.176$

The frequency Response is given by

$$\bar{H}(e^{j\omega}) = \sum_{n=0}^{\frac{N-1}{2}} a_n \cos\omega n.$$

$$a(0) = h\left(\frac{N-1}{2}\right)$$

$$a(n) = 2h\left(\frac{N-1}{2} - n\right).$$

$$H(e^{j\omega}) = \sum_{n=0}^{\frac{N-1}{2}} 2h\left(\frac{17-1}{2} - n\right) \cos \omega n .$$

$$= 2h(8)\cos\omega(0) + 2h(7)\cos\omega + 2h(6)\cos 2\omega$$
$$+ 2h(5)\cos 3\omega + 2h(4)\cos 4\omega + \dots + 2h(3)\cos 5\omega$$
$$+ 2h(2)\cos 6\omega + 2h(1)\cos 7\omega$$
$$+ 2h(0)\cos 8\omega$$

$$= 0.352 + 0.336\cos\omega + 0.29\cos 2\omega + 0.22\cos 3\omega$$
$$+ 0.1396\cos 4\omega + 0.052\cos 5\omega - 0.0272\cos 6\omega$$
$$- 0.0824\cos 7\omega - 0.112\cos 8\omega$$

ω in degrees	0	15	30	60	75	105
$H(e^{j\omega})$	0.9286	+0.239	0.231	0.24	0.245	.247

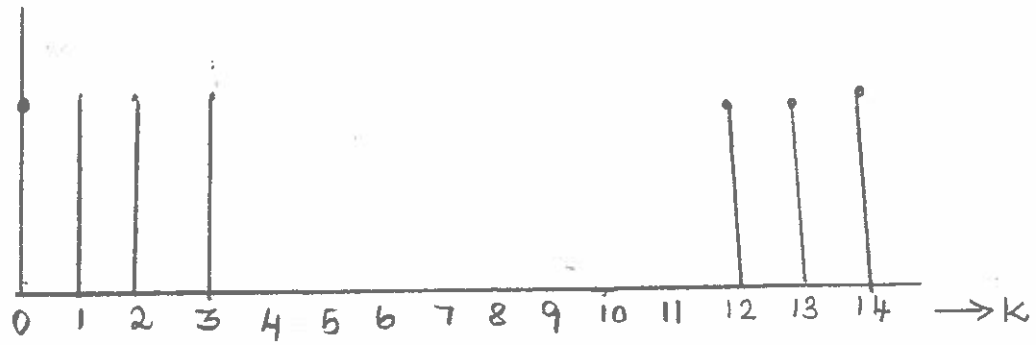
Determine the co-efficients of a linear phase FIR filter of length $M = 15$ has a symmetric unit sample response and a frequency response that satisfies the conditions.

$$H\left(\frac{2\pi K}{15}\right) = 1 \qquad K = 0, 1, 2, 3.$$
$$= 0 \qquad K = 4, 5, 6, 7.$$

Solution

$|H(k)| = 1$ for $0 \le k \le 3$ and $12 \le k \le 14$

$= 0$ for $4 \le k \le 11$



$$\theta(k) = -\left(\frac{N-1}{N}\right)\pi k = \frac{-14}{15}\pi k \qquad 0 \le k \le 7$$

and $\theta(k) = 14\pi - \frac{14\pi k}{15}$ $\qquad 8 \le k \le 14$

$H(k) = e^{-j14\pi k/15}$ for $k = 0, 1, 2, 3.$

$= 0$ for $4 \le k \le 11$

$$h(n) = \frac{1}{N}\left[H(0) + 2\sum_{k=1}^{\frac{N-1}{2}} Re\left(H(k)\, e^{j2\pi nk/15}\right)\right]$$

$$= \frac{1}{15}\left[1 + 2\sum_{k=1}^{7} Re\left(e^{-j14\pi k/15}\, e^{j2\pi nk/15}\right)\right]$$

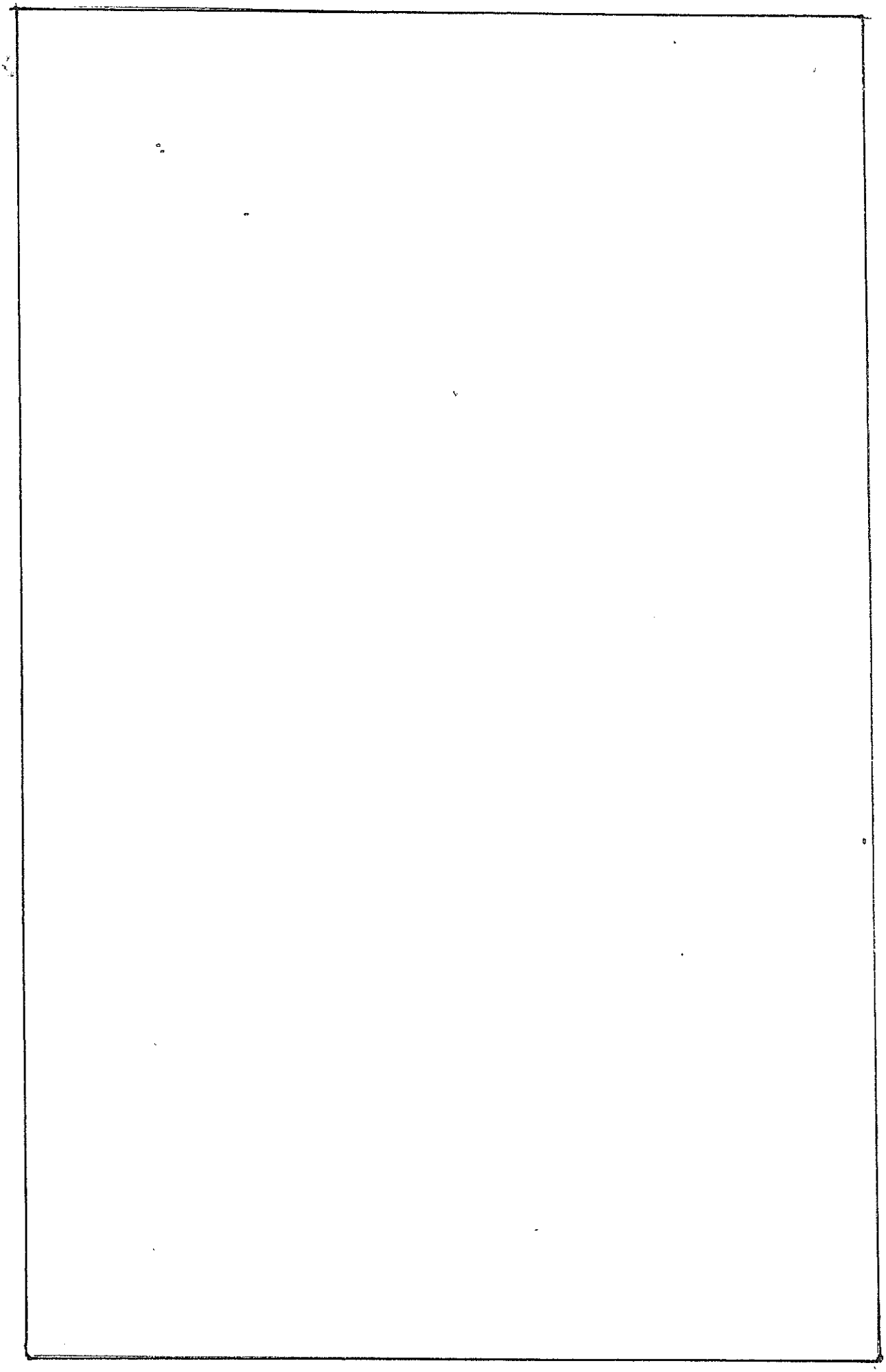
$$= \frac{1}{15}\left[1 + 2\sum_{k=1}^{3} \cos\frac{2\pi k(7-n)}{15}\right]$$

$$= \frac{1}{15}\left[1 + 2\cos\frac{2\pi(7-n)}{15} + 2\cos\frac{4\pi(7-n)}{15} + 2\cos\frac{6\pi(7-n)}{15}\right]$$

$h(0) = h(14) = -0.05$ $\quad h(1) = h(13) = 0.041$ $\quad h(4) = h(10) = -0.1078$

$h(2) = h(12) = 0.066$ $\quad h(3) = h(11) = -0.0866$ $\quad h(5) = h(9) = 0.034$

$h(6) = h(8) = 0.3188$ $\quad h(7) = 0.466.$



UNIT - IV

FINITE WORD LENGTH EFFECTS.

Part A (2 Marks).

1. What do you understand by input quantization error? Dec 13.

* In DSP, the Continuous time input signals are Converted in to digital using a 'b' bit ADC.
* The representation of continuous signal amplitude by a fixed digit produces an error, which is known as input quantization error.

2. What is product round off Noise? (or) What is product quantization error?

*It arises at the output of a multiplier. Multiplication of b bit data with a b bit Co-efficient results a product having 2b bits.

*Since a 'b' bit register is used, the multiplier output must be rounded or truncated to 'b' bits which produce Noise. This noise is known as product quantization error (or) product round off Noise

3. What are the quantization method employed in digital systems. June 2013

1. Truncation
2. Rounding.

4. Compare truncation and Rounding error.

Truncation	Rounding
* Truncation is the Process of reducing the size of binary number by discarding all bits less significant than least significant bit that is retained.	* Rounding is the process of reducing the size of a binary number to finite word size of bits such that the rounded b bit number is closet to the orginal unquantized number.
* Truncation of binary number b bits all the less significant bits beyond b^{th} bit are discarded.	* Rounding of a binary number b bits first truncated the unquantized number to 'b' bits then zero (or) one is added to the LSB.

5. What is dead band of a filter. Dec 12

* The limit cycles occurs are result of quantization effect in multiplication
* The amplitudes of the output during Limit cycle are confined to a range of values.
* These range of values are called deadband of the filter.

$$\text{Dead band} = \frac{\frac{1}{2} 2^{-b}}{1-|\alpha|}$$

6. What is Limit cycle oscillation?

In Recursive system when the input is zero or some non zero constant value, the non-linearities due to finite precision arithmetic operation may cause periodic oscillations in the output. The oscillations are called limit-cycles.

7. Define Zero input limit cycle oscillations. June 13

* If the system output enters a limit cycle, it will continue to remain in limit cycle even when the input is made zero.

* these limit cycles are also called zero input limit cycle oscillations.

8. What is the effect of quantization on pole location?

* Quantization of co-efficients in digital filters lead to slight changes in their values.

* the change in value of filters co-efficients modifies the pole locations.

* Some times the pole locations will be changed in such a way that the system may derive into instability.

9. State the methods used to prevent overflow

Dec 13

* Saturation arithmetic
* Scaling.

10. What is known as overflow oscillations.

* The addition of two fixed point arithmetic numbers cause overflow when the sum exceeds the word size available to store the sum.

* This overflow caused by adder make the filter output to oscillate between maximum amplitude limits.
* Such limit cycles have been referred to as overflow oscillations.

UNIT IV

Part : B

1. Explain Fixed point and Floating point representations

Types of Number Representation :- 1. Fixed Point
2. Floating Point.

(i) Fixed point Number Representation : Apr|may 2015

* In Fixed point, the position of the binary point is fixed. ~~The bit to~~

* It has 3 different formats for representing negative binary fraction number.

(a) sign - magnitude format
(b) One's Complement format
(c) Two's Complement format.

(a) Sign magnitude format :

In signmagnitude format the -ive value of a given number differ only in signbit. The sign digit is zero for +ive number and one for -ive numbers.

Example :

$(1.25)_{10} \longrightarrow (\underline{0}1.01)_2$

$(-1.25)_{10} \longrightarrow (\underline{1}1.01)_2$

$+0.125 \longrightarrow +.001 \xrightarrow[\text{signbit}]{\text{Append}} 0.001 \xrightarrow[\text{dot}]{\text{Remove}} 0001_2$

$-0.125 \longrightarrow -.001 \longrightarrow 1.001 \longrightarrow 1001_2$

Ones Complement Format:

For +ive numbers the 1's complement format and sign magnitude format are identical. For -ive number the 1's complement generated by changing all the zeros to ones and ones to zeros in the sign magnitude representation of its absolute value.

$(1.25)_{10} \Longrightarrow (01.01)_2$

$(-1.25)_{10} \Longrightarrow (10.10)_2$

Two's Complement Format:

In two's complement format the -ive of the given number is obtained by taking 1's complement of its +ive value representation and then adding one to the LSB.

Ex:

$+.125 \xrightarrow[\text{Binary}]{\text{Conv}} +0.01 \xrightarrow[\text{signbit}]{\text{Add}} 0.001 \xrightarrow[\text{dot}]{\text{Remove}} 0001$

$-.125 \xrightarrow[\text{Binary}]{\text{Conv}} -.001 \xrightarrow[\text{fraction part}]{\text{complement}} -.110 \xrightarrow[\text{LSB}]{\text{add 1}} -.111$

$1.111 \xrightarrow{\text{add sign}} 1111$

Floating Point Representation

June 13.

The floating point representation is employed to represent larger range of number in a given binary word size.

The floating point number is represented as

$$F = M \times 2^E$$

Where M is Mantissa $(0.5 \leq M < 1)$

E is Exponent Part.

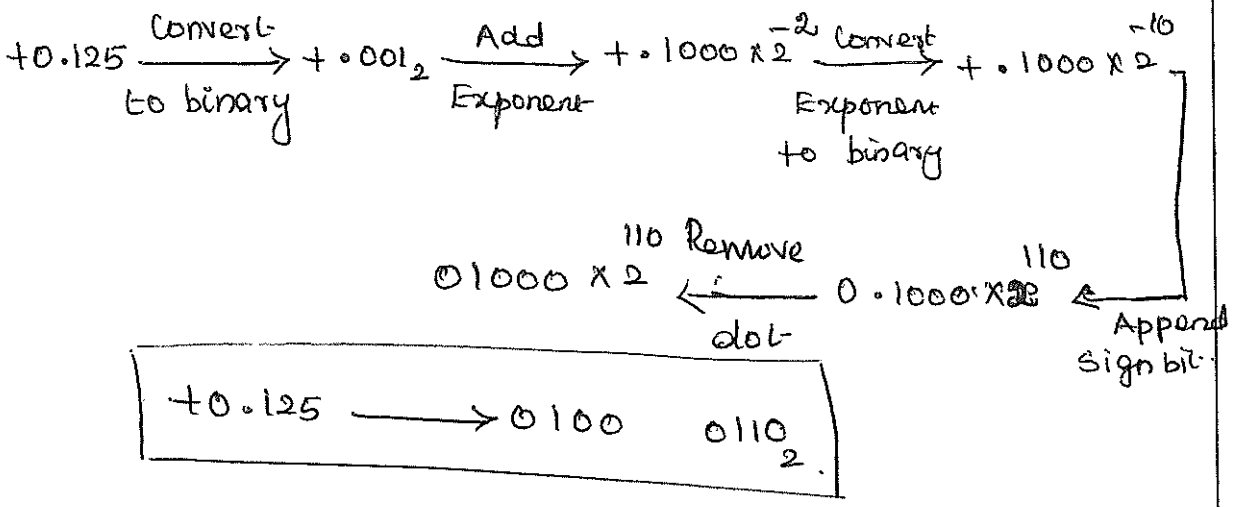
The floating point representation is explained by considering a 5 bit mantissa and three bit exponent with a total data size of 8 bits.

In mantissa the leftmost bit is used to represent the sign and other four bits are used to represent a binary fraction number. In exponent the leftmost it is used to represent the sign and the other two is are used to represent a binary integer Number.

Example:

+5 $\xrightarrow[\text{to binary}]{\text{Convert}}$ +101 $\xrightarrow{\text{add exponent}}$ $+.1010 \times 2^{+3}$ $\xrightarrow[\text{Exponent to binary}]{\text{Convert}}$ $+.1010 \times 2^{11}$

$+.1010 \times 2^{11}$ → Append sign bit → 0.1010×2^{011} → Remove dot → 01010×2^{011}



+0.125 ⟶ 0100 0110$_2$.

2. Derive the expression for Quantization Noise Power:.

Input Noise Power.

The error signal e(n) has the following properties.

1. The error signal e(n) is a Stationary Random process.

2. The error sequence is uncorrelated with x(n)

3. The error is a white noise process with uniform amplitude probablity distribution. over the range of Quantization error.

The probability density function for error signal is given by

$$P(e) = \frac{1}{2^{-b}} \; ; \; \frac{-2^{-b}}{2} \leq e \leq \frac{2^{-b}}{2}$$

The power of the quantization noise is equal to the Variance of the error.

The power of the Quantization noise is equal to the variance of the error signal $\hat{e}(n)$ is given by

$$\sigma_e^2 = E[e^2(n)] - [E[e(n)]^2$$

Since $e(n)$ is uncorrelated the mean value

$$E[e(n)] = 0.$$

$$\sigma_e^2 = E[e^2(n)]$$

$$\sigma_e^2 = \int e^2 P(e)\, de$$

$$= \int_{-2^{-b}/2}^{2^{-b}/2} e^2 \frac{1}{2^{-b}}\, de$$

$$= \frac{1}{2^{-b}} \left[\frac{e^3}{3}\right]_{-2^{-b}/2}^{2^{-b}/2}$$

$$= \frac{1}{2^{-b} \times 3} \left[\left(\frac{2^{-b}}{2}\right)^3 - \left(\frac{-2^{-b}}{2}\right)^3\right]$$

$$= \frac{1}{2^{-b} \times 3} \left(\frac{2^{-b} + 2^{-b}}{8}\right) = \frac{2 \cdot 2^{-3b}}{24 \times 2^{-b}}$$

$$\boxed{\sigma_e^2 = \frac{2^{-2b}}{12}}$$

(ii) Steady State Output Noise Power

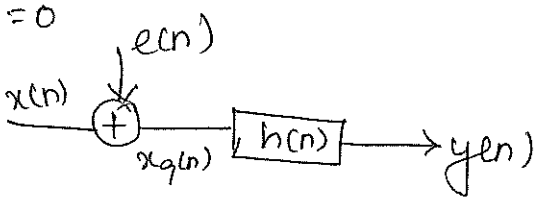
After converting the CT signal into Digital signal that quantized signal is applied to a digital system with system function H(z)

After Quantization we have quantization noise power σ_e^2 as a input noise power of the digital system is given by.

$$\sigma_{e_0}^2 = \sigma_e^2 \text{ system power gain}$$

$$\sigma_{e_0}^2 = \sigma_e^2 \sum_{n=0}^{\infty} h^2(n) \quad \text{——— (1)}$$

$$Z[h(n)] = \sum_{n=0}^{\infty} h(n) z^{-n}.$$



$$Z[h^2(n)] = \sum_{n=0}^{\infty} h^2(n) z^{-n}$$

$$= \sum_{n=0}^{\infty} h(n) h(n) z^{-n} \quad \text{——— (2)}$$

Take Inverse Z transform

$$h(n) = \frac{1}{2\pi j} \oint H(z) z^{n-1} dz. \quad \text{——— (3)}$$

5.5 eq (3) & (2)

$$Z\left[h^2(n)\right] = \sum_{n=0}^{\infty} h(n)\left[\frac{1}{2\pi j}\oint H(z)\, z^{n-1}\, dz\right] z^{-n}.$$

$$Z\left[h^2(n)\right] = \frac{1}{2\pi j}\oint H(z) \sum_{n=0}^{\infty} h(n)\, z^{n-1}\, z^{-n}\, dz.$$

$$Z\left[h^2(n)\right] = \frac{1}{2\pi j}\oint H(z) \sum_{n=0}^{\infty} h(n)\, \frac{dz}{z}$$

$$\sum_{n=0}^{\infty} h^2(n)\, z^{-n} = \frac{1}{2\pi j}\oint H(z) \sum_{n=0}^{\infty} \frac{dz}{z}.$$

$$\sum_{n=0}^{\infty} h^2(n) = \frac{1}{2\pi j}\oint H(z) \sum_{n=0}^{\infty} h(n)\, z^{n}\, \frac{dz}{z}.$$

$$= \frac{1}{2\pi j}\oint H(z) \sum_{n=0}^{\infty} h(n)\left(z^{-1}\right)^{-n} \frac{dz}{z}.$$

$$\sum_{n=0}^{\infty} h^2(n) = \frac{1}{2\pi j}\oint H(z)\, H(z^{-1})\, \frac{dz}{z} \qquad \text{---------- (4)}$$

Substitute (4) in eq (1).

$$\boxed{\sigma_{e0}^2 = \sigma_e^2 \frac{1}{2\pi j}\oint H(z)\, H(z^{-1})\, \frac{dz}{z}}$$

3. The Output of A to D converter is applied to a digital filter with system function $H(z) = \frac{0.5z}{z-0.5}$. Determine output noise power when the input signal is quantized to 8-bits.

Soln:

Given $b = 8$ bits.

$$H(z) = \frac{0.5z}{z-0.5}.$$

Let $z = z^{-1}$

$$H(z^{-1}) = \frac{0.5z^{-1}}{z^{-1}-0.5}.$$

The output Noise power

$$\sigma_{eo}^2 = \sigma_e^2 \frac{1}{2\pi j} \oint H(z)\, H(z^{-1}) \frac{dz}{z}.$$

$$\sigma_{eo}^2 = \sigma_e^2 \frac{1}{2\pi j} \oint \left(\frac{0.5z}{z-0.5}\right)\left(\frac{0.5z^{-1}}{z^{-1}-0.5}\right)\frac{dz}{z}.$$

$$= \sigma_e^2 \frac{1}{2\pi j} \oint \left(\frac{0.5z}{z-0.5}\right)\left(\frac{0.5z^{-1}}{z^{-1}(1-0.5z)}\right)z^{-1}\, dz$$

$$= \sigma_e^2 \frac{1}{2\pi j} \oint \frac{0.5z}{(z-0.5)} \cdot \frac{0.5z^{-1}}{z^{-1}(1-0.5z)} \cdot \frac{dz}{z}.$$

$$\sigma_{e_0}^2 = \sigma_e^2 \frac{1}{2\pi j} \oint \frac{0.25}{(z-0.5)(1-0.5z)} \, dz .$$

Here the poles $z = 0.5$

$$1 - 0.5z = 0$$

$$0.5z = 1$$

$$z = \frac{1}{0.5} = 2 .$$

$\boxed{z = 0.5}$, $\boxed{z = 2}$

$$\sigma_{e_0}^2 = \sigma_e^2 \left[\sum \text{Residue of } \frac{0.25}{(z-0.5)(1-0.5z)} \text{ at the poles } z=0.5, \; z=2 \right]$$

Since $z = 2$ lies outside of the unit circle so residue values is zero.

$$\sigma_{e_0}^2 = \sigma_e^2 \left[\text{Residue of } \frac{0.25}{(z-0.5)(1-0.5z)} \text{ at } z = 0.5 \right]$$

$$= \sigma_e^2 \left(z - 0.5 \right) \left(\frac{0.25}{(z-0.5)(1-0.5z)} \right) \Bigg|_{z=0.5}$$

$$= \sigma_e^2 \frac{0.25}{1 - 0.5 \times 0.5} = \frac{\sigma_e^2 \, 0.25}{1 - 0.25}$$

$$\boxed{\sigma_{e_0}^2 = \sigma_e^2 \; 0.33}$$

$$\sigma_{e_0}^2 = \frac{2^{-2b}}{12} 0.33$$

$$= \frac{2^{-2\times 8}}{12} \times 0.33.$$

$$= \frac{2^{-16}}{12} \times 0.33$$

$$\sigma_{e_0}^2 = 1.271 \times 10^{-6}$$

$$\sigma_{e_0}^2 = 0.4196 \times 10^{-6}.$$

$$\boxed{\sigma_{e_0}^2 = 4.196 \times 10^{-7}}$$

4. Explain the characteristics of a limit cycle oscillation with respect to the system, described by the equation $y(n) = 0.95\,y(n-1) + x(n)$. When the product is quantized to 5 bits by rounding and the system is excited by the input $x(n) = \begin{cases} 0.75 & \text{for } n = 0 \\ 0 & \text{for } n \neq 0 \end{cases}$. June 13

Soln:

Given:

$$y(n) = 0.95\,y(n-1) + x(n).$$

$$x(n) = \begin{cases} 0.75 & \text{for } n = 0 \\ 0 & n \neq 0. \end{cases}$$

$$y(n) = Q\left[0.95\, y(n-1)\right] + x(n)$$

$n = 0$

$$y(0) = Q\left[0.95 \times y(-1)\right] + x(0)$$

$$= Q\left[0.95 \times 0\right] + 0.75$$

$$\boxed{y(0) = 0.75}$$

$n = 1$

$$y(1) = Q\left[0.95 \times y(0)\right] + x(1)$$

$$= Q\left[0.95 \times 0.75\right] + 0$$

$$= Q\left[0.7125\right]$$

$$= \left[0.101101\ldots\right]$$

$$= 0.1011$$

$$\boxed{y(1) = 0.6875}$$

$n = 2$

$$y(2) = Q\left[0.95 \times y(1)\right] + x(2)$$

$$= Q\left[0.95 \times 0.6875\right]$$

$$= Q\left[0.6531\right]$$

$$= Q\left[0.10100\ldots\right]$$

$$= 0.1010$$

$$\boxed{y(2) = 0.625}$$

$n = 3$

$$y(3) = Q\left[0.95 \times y(2)\right] + x(3)$$

$$= Q\left[0.95 \times 0.625\right]$$

$$= Q\left[0.59375\right]$$

$$= Q\left[0.1001\ldots\right]$$

$$= 0.1010$$

$$\boxed{y(3) = 0.625}$$

Dead Band

$$= \frac{\frac{1}{2}\, 2^{-b}}{1-a}$$

$$= \frac{\frac{1}{2}\, 2^{-4}}{1-0.95}$$

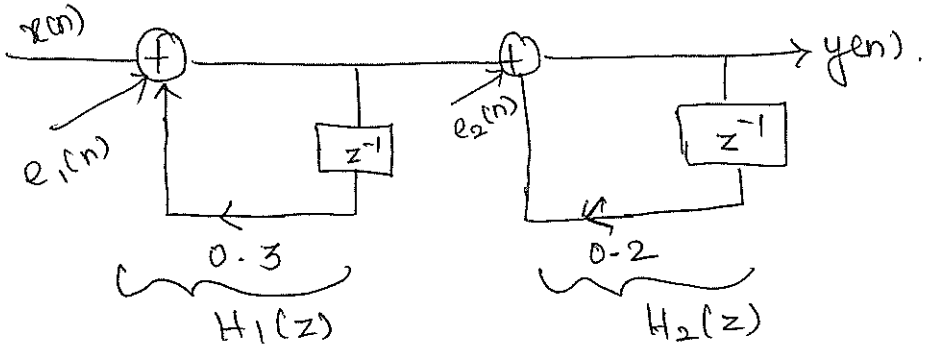
$$\boxed{DB = 0.625}$$

5. A Cascade structure has a two individuals section $H_1(z) = \frac{1}{1-0.3z^{-1}}$ and $H_2(z) = \frac{1}{1-0.2z^{-1}}$. Determine the overall output Noise power by product Quantization Noise model.

April 2015

Soln:

The Product Quantization noise model for the given system.



Let $H(z) = H_1(z) \cdot H_2(z)$

$$H(z) = \frac{1}{(1-0.3z^{-1})(1-0.2z^{-1})}$$

The Noise transfer function seen by $e_1(n)$ is $H(z) = H_1(z)$ and the Noise transfer function seen by $e_2(n)$ is $H_2(z)$.

Hence the overall output Noise power is

$$\sigma_{eo}^2 = \sigma_{e_1}^2 + \sigma_{e_2}^2$$

$\sigma_{e_1}^2 \longrightarrow$ Noise power due to $e_1(n)$

$\sigma_{e_2}^2 \longrightarrow$ Noise power due to $e_2(n)$

The output Noise power due to $e_1(n)$ is given by

$$\sigma_{e_1}^2 = \sigma_e^2 * \frac{1}{2\pi j} \oint H(z)\, H(z^{-1}) \cdot \frac{dz}{z}.$$

$$= \sigma_e^2 * \frac{1}{2\pi j} \oint \frac{1}{(1-0.3z^{-1})(1-0.2z^{-1})} \times \frac{1}{(1-0.3z)(1-0.2z)} \frac{dz}{z}.$$

$$= \sigma_e^2 \frac{1}{2\pi j} \oint \frac{1}{z^{-1}(z-0.3)\, z^{-1}(z-0.2)(1-0.3z)(1-0.2z)} \times \frac{dz}{z}.$$

$$= \sigma_e^2 \frac{1}{2\pi j} \oint \frac{1}{z^{-1}(z-0.3)\, z^{-1}(z-0.2)(1-0.3z)(1-0.2z)} \frac{dz}{z}$$

$$= \sigma_e^2 \frac{1}{2\pi j} \oint \frac{z}{(z-0.3)(z-0.2)(1-0.3z)(1-0.2z)} \cdot dz$$

here the poles $z = 0.3$, $z = 0.2$ lies inside of unit circle.

hence

$$\sigma_{e_1}^2 = \sigma_e^2 \left[\text{Residue at } z=0.3 + \text{Residue at } z=0.2 \right]$$

$$= \sigma_e^2 \left\{ \left[(z-0.3) \frac{z}{(z-0.3)(z-0.2)(1-0.3z)(1-0.2z)} \right] \right.$$

$$\left. + (z-0.2) \frac{z}{(z-0.3)(z-0.2)(1-0.3z)(1-0.2z)} \right]_{z=0.2}^{z=0.3} \Bigg\}$$

$$= \sigma_e^2 \left(3.507 - 2.126 \right)$$

$$\boxed{\sigma_{e_1}^2 = \sigma_e^2 \; 1.291}$$

The Output Noise power due to $e_2(n)$ is.

$$e_2(n) = \sigma_e^2 \frac{1}{2\pi j} \oint H_2(z) \, H_2(z^{-1}) \cdot \frac{dz}{z}$$

$$= \sigma_e^2 \frac{1}{2\pi j} \oint \frac{1}{(1-0.2z^{-1})(1-0.2z)} \frac{dz}{z}.$$

$$= \sigma_e^2 \frac{1}{2\pi j} \oint \frac{1}{z^{-1}(z-0.2)(1-0.2z)} \frac{dz}{z}$$

here the poles $z = 0.2$ lies inside of unit circle.

$$\sigma_{e2}^2 = \sigma_e^2 \left[\text{Residue at } z = 0.2 \right]$$

$$= \sigma_e^2 \left[(z-0.2) \frac{1}{(z-0.2)(1-0.2z)} \Big|_{z=0.2} \right]$$

$$= \sigma_e^2 \left(\frac{1}{1 - 0.2 \times 0.2} \right)$$

$$\boxed{\sigma_{e_2}^2 = \sigma_e^2 \; 1.042.}$$

The overall output Noise power

$$\sigma_{eo}^2 = \sigma_{e1}^2 + \sigma_e^2 \qquad \sigma_e^2 = \frac{2^{-2b}}{}$$

$$= \sigma_e^2 \, 2.83$$

$$\sigma_{eo}^2 = 0.194 \times 2^{-2b}.$$

6. Consider a second order FIR Filter with system transfer function $H(z) = \frac{1}{1 - 0.85z^{-1} + 0.175z^{-2}}$.

Determining the effect of Quantization on pole location of the given system function is (i) Direct form (ii) Cascade form. with b = 4 bits.

Dec 12

Soln:-

1. Direct form:

$$H(z) = \frac{1}{1 - 0.85z^{-1} + 0.175z^{-2}}$$

Let us Quantize the Co-efficient in to 4 bits.

$0.85 \xrightarrow{\text{Convert to Binary}} 0.110110 \xrightarrow{\text{Quantized to 4 bits}} 0.1101 \xrightarrow{\text{Convt to decimal}} 0.8125$

$0.175 \xrightarrow{\text{Convert to Binary}} 0.00101 \xrightarrow{\text{Quantized to 4 bits}} 0.0010 \xrightarrow{\text{Convert to decimal}} 0.125.$

$$[H(z)]_q = \frac{1}{1 - 0.8125z^{-1} + 0.125z^{-2}}$$

2. Cascade form

$$H(z) = \frac{1}{1 - 0.85z^{-1} + 0.175z^{-2}}$$

$$H(z) = \frac{z^2}{z^2 - 0.85z + 0.175}$$

$$= \frac{z^2}{(z-0.5)(z-0.35)}$$

$$H(z) = \frac{1}{(1-0.5z^{-1})(1-0.35z^{-1})}$$

Let us truncate the co-efficients.

$0.5 \xrightarrow[\text{to binary}]{\text{Convert}} 0.10 \xrightarrow[\text{to 4 bits}]{\text{Quantized}} 0.10 \xrightarrow[\text{to decimal}]{\text{Convert}} 0.5$

$0.35 \xrightarrow[\text{binary}]{\text{Convert to}} 0.010110 \xrightarrow[\text{to 4 bits}]{\text{Quantized}} 0.0101 \xrightarrow[\text{decimal}]{\text{Convert}} 0.3125$

The Quantized system function is given by

$$[H(z)]_q = \frac{1}{(1-0.5z^{-1})(1-0.3125z^{-1})}$$

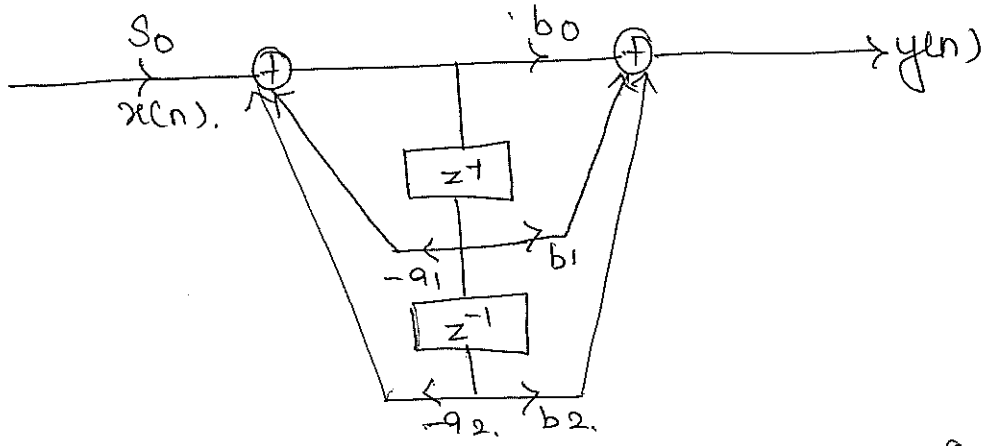
here the poles are changed from actual position.

7. Signal Scaling: June 13, Dec 12

In order to limit the amount of non linear distortion it is important to scale the input signal and the unit sample response

between the input and any internal summing node in the system that overflow becomes rare event.

Let us consider II nd order FIR filter shown in Fig



A scaling factor s_0 is introduced between input x(n) and the adder I to prevent the overflow at the output adder I. The scaling factor is expressed as,

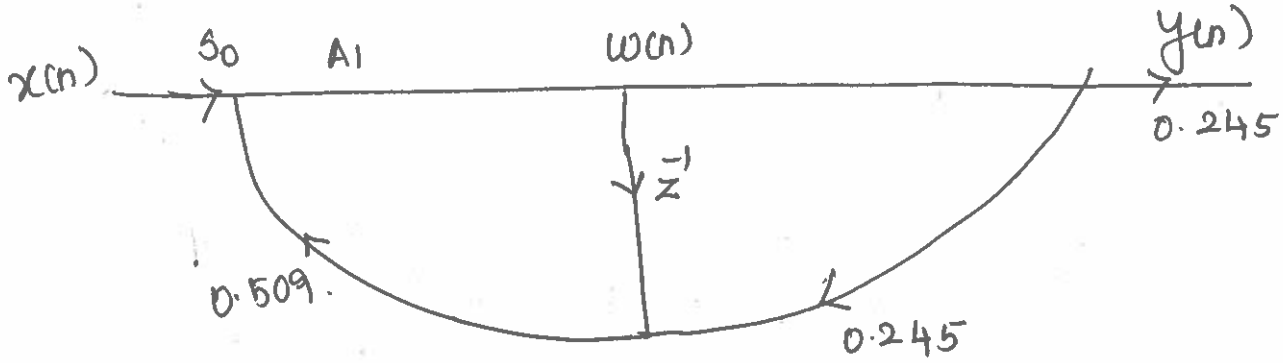
$$s_0^2 = \frac{1}{\frac{1}{2\pi j}\oint \frac{1}{D(z)D(z^{-1})}\frac{dz}{z}}$$

Where $$I = \frac{1}{2\pi j}\oint \frac{1}{D(z)(D(z^{-1})}\frac{dz}{z}.$$

$$\therefore s_0^2 = \frac{1}{I}.$$

$$\boxed{s_0 = \sqrt{\frac{1}{I}}}$$

For the digital Network shown in fig find H(z) and Scale factor. So to avoid overflow regarding A_1.



Solution:-

$$H(z) = \frac{0.245\, z^{-1}}{1 + 0.509\, z^{-1}}$$

$$S^2 = \frac{1}{2\pi j} \oint_c \frac{z^{-1}\, dz}{D(z)D(z^{-1})}$$

$$D(z) = 1 + 0.509\, z^{-1}$$

$$D(z^{-1}) = 1 + 0.509 z.$$

$$\frac{1}{2\pi j} \oint_c \frac{z^{-1}\, dz}{D(z)\, D(z^{-1})} = \frac{1}{2\pi j} \oint_c \frac{z^{-1}\, dz}{(1+0.509 z^{-1})(1+0.509 z)}$$

$$= \frac{1}{2\pi j} \oint_c \frac{z^{-1}\, dz}{z^{-1}(z + 0.509) + (1 + 0.509 z)}$$

Find the residue of $\dfrac{z^{-1}}{(1+0.509z^{-1})(1+0.509z)}$

The residue due to pole $z = \dfrac{1}{0.509}$ is zero (exceeds the unit circle).

The residue due to pole $z = 0.509$ is given by.

$$z + (0.509)\left.\frac{1}{(z+0.509)(1+0.509z)}\right|_{z=-0.509}$$

$$= 1.3496.$$

$$\therefore s^2 = \frac{1}{1.3496.} = 0.740$$

$$\boxed{\therefore s = 0.86}$$

UNIT - V

DSP APPLICATIONS

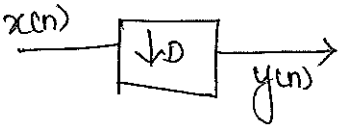
PART - A (2 Marks)

1. What is multirate signal processing?

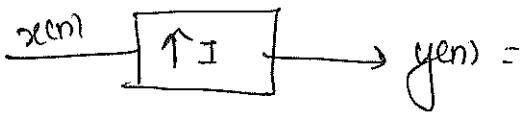
* The Processing of discrete time signals of different sampling rate in different parts of a system is called Multirate signal processing.

2. What is meant by downsampling (Decimation) and upsampling (Interpolation)? APR/May 2015 Dec 12

* Decimation is the process of reducing the samples of the discrete time signal by an integer factor D.



* Interpolation (or) upsampling is the process of increasing the samples of the discrete time signal by an integer factor I. $y(n) = x\left(\frac{n}{I}\right)$



3. State Nyquist Sampling Theorem?

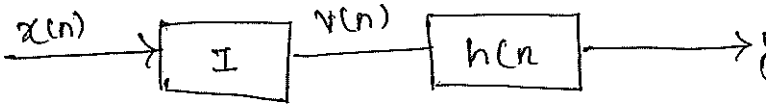
Sampling theorem states that- 'a band limited signal $x(t)$ is uniquely determined from its samples if the sampling frequency is greater than (or) equal to twice the maximum frequency of the signal.

$$f_s \geq 2 f_m.$$

4 What is anti imaging filter? June 13

The lowpass filter is used at the output of an interpolator is called anti imaging filter.

It is used to eliminate the multiple images in the output spectrum of the interpolator

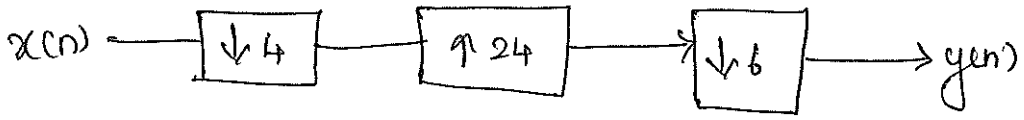


5. What are called polyphase filters?

* The polyphase decomposition is dividing an Nth order filter into L sections of sub filters that can be realized in parallel.

* In this decomposition the sub filters will differ only in phase characteristics and so they are called polyphase filters.

6. Find the expression for the following multirate System.



1. $y(n) = x(4n)$
2. $y(n) = x\left(\frac{4n}{24}\right)$
3. $y(n) = x\left(\frac{4(6n)}{24}\right)$
4. $y(n) = x(n)$

7. What is subband coading?

*Sub band Coding is the process of split the input signal x(n) into many narrow band signals to improve the efficiency of transmission with limited bandwidth.

8. State the various application of DSP.

* Telecommunication
* Consumer Electronics.
* Instrumentation and Control.
* Image Processing
* Medicine.
* speech procening

9. Write any four applications of Multirate signal processing. Dec 13, June 13

* Sub-band Coading of speech signal and image compression.
* QMF. (Quadrature Mirror Filters) for realizing alias free LTI Multirate system.
* Narrowband FIR and IIR filters for Various applications.
* Digital transmultiplexers for converting TDM signals to FDM. signals and Vice versa.

10. Sampling Rate Conversion

The procen of converting a signal from a sampling rate to another sampling rate is called sampling rate conversion

1. Decimation
2. Interpolation.

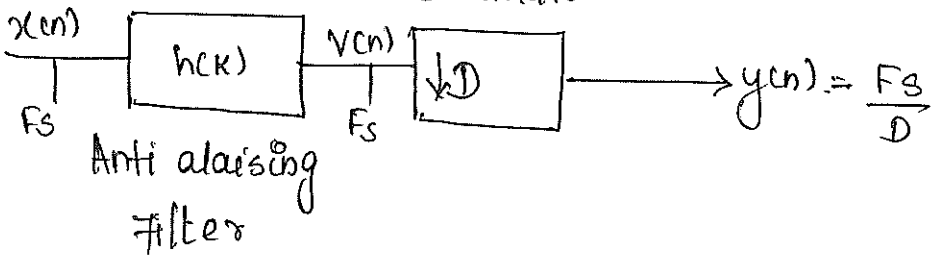
UNIT V

Part - B.

1. Derive the spectrum of down sampled and up sampled signal. Apr/May 2015.

(i) Down sampled signal.

Decimator.



The block diagram shows the process of decimating a signal x(n) by an integer factor D. The decimator also known as down sampler, sub sampler. It consist of anti alasing filter(k), it use to reduce the over lapping of the signal.

Spectrum of Down sampler:

Let us consider the input signal x(n) and output- y(n).

The downsampled signal y(n) is obtained by multiplying the sequence x(n) with a periodic train of pulses p(n) with a period D and then is given by

$$P(n) = \begin{cases} 1 & n = 0 \; ; \pm D \; \pm 2D \\ 0 & 0.W \end{cases}$$

The Inverse Discrete Fourier transform of $P(n)$ is given by

$$p(n) = \frac{1}{D} \sum_{k=0}^{D-1} P(k)\, e^{j 2\pi k n / D}.$$

$$= \frac{1}{D} \sum_{k=0}^{D-1} e^{j 2\pi k n / D} \qquad P(k) = 1.$$

Let $x'(n)$ is the sequence of multiplying the sequence $x(n)$ and $P(n)$.

$$x'(n) = x(n) \cdot P(n).$$

So the output $y(n) = x'(nD)$.

The z transform of output sequence is given by

$$Y(z) = \sum_{n=-\infty}^{\infty} y(n)\, z^{-n}$$

$$= \sum_{n=\infty}^{\infty} x'(nD)\, z^{-n}$$

Put $m = nD \; ; \; n = m/D$

$$y(z) = \sum_{m=-\infty}^{\infty} x'(m)\, z^{-m/D}.$$

$$\therefore$$

Let $m = n$ $\quad y(z) = \sum_{n=-\infty}^{\infty} x'(n)\, z^{-n/D}.$

$$= \sum_{n=-\infty}^{\infty} x(n) P(n)\, z^{-n/D}. \qquad x'(n) = x(n) P(n).$$

3.5 $P(n)$ Value

$$= \sum_{n=-\infty}^{\infty} x(n) \frac{1}{D} \sum_{k=0}^{D-1} e^{j 2\pi k n / D}\, z^{-n/D}.$$

$$= \frac{1}{D} \sum_{k=0}^{D-1} \sum_{n=-\infty}^{\infty} x(n) \left[e^{-j2\pi k/D} z^{1/D} \right]^{-n}$$

$$= \frac{1}{D} \sum_{k=0}^{D-1} \left(e^{-j2\pi k/D} z^{1/D} \right)$$

Let $z = e^{j\omega}$

$$Y(e^{j\omega}) = \frac{1}{D} \sum_{k=0}^{D-1} e^{-j2\pi k/D} e^{j\omega/D}.$$

$$Y(e^{j\omega}) = \frac{1}{D} \sum_{k=0}^{D1} e^{-j2\pi k/D} e^{j\omega/D}.$$

$$\boxed{Y(e^{j\omega}) = \frac{1}{D} \sum_{k=0}^{D-1} e^{j(\omega-2\pi k)/D}}$$

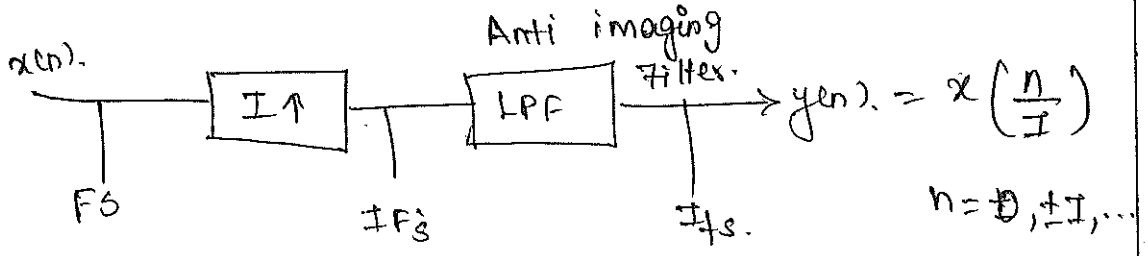
On expanding the equation

$$Y(e^{j\omega}) = \frac{1}{D} \times e^{j\omega/D} + \frac{1}{D} \times \left[e^{j(\omega-2\pi)/D} \right] + \cdots$$

$$+ \frac{1}{D} \left[e^{j(\omega-2\pi(D-1))/D} \right].$$

1. The term $X(e^{j\omega/D})$ is the frequency stretched version of $X(e^{j\omega})$ by factor D.
2. The general term $X(e^{j(\omega-2\pi k)/D}$ is the shifter version of $X(e^{j\omega/D})$, shifted by a factor $2\pi k$.
3. Also it is obtained that the magnitude of each components of the spectrum is scaled by a factor D.

(ii) Upsampling (or) Interpolation.



It consist of a sampling rate expander, represented by up arrow, and the LPF is used to remove the repeated signals.

Spectrum of the up sampled signal:-

The output of upsampled signal

$$y(n) = x\left(\frac{n}{I}\right) \quad n = 0, \pm I, \pm 2I \ldots.$$

The z transform of the signal y(n) is given by $y(z) = \sum_{n=-\infty}^{\infty} y(n) \cdot z^{-n}$.

$$\text{S.S. } y(n) \quad y(z) = \sum_{n=-\infty}^{\infty} x\left(\frac{n}{I}\right) z^{-n}.$$

Put $m = \frac{n}{I}$ $\quad n = mI$.

$$Y(z) = \sum_{m=-\infty}^{\infty} x\left(\frac{mI}{I}\right) z^{-mI}$$

Let $m = n$.

$$y(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-nI}.$$

$$= \sum_{n=-\infty}^{\infty} x(n)(z^I)^{-n}$$

$$Y(z) = X(z^I)$$

put $z = e^{j\omega}$

$$Y(e^{j\omega}) = X(e^{j\omega I}).$$

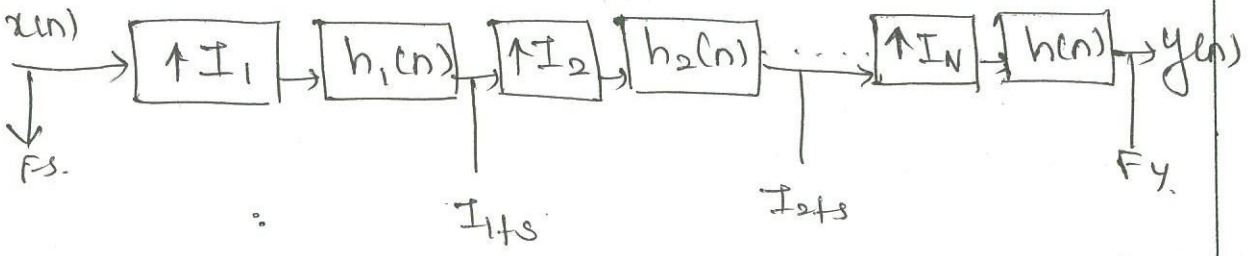
2.(i) 2. Discuss the steps involved in the implementation of multistage sampling rate conversion. Dec 2013 June 13 Dec 12

If the decimation factor D and interpolation factor I are much larger than unity, the implementation of sampling rate conversion in a single stage is computational efficient. Therefore for performing sampling rate conversion for either $D >> I$ (or) $I >> 1$ we go for multi stage implementation.

If the interpolation factor $I >> 1$, then we express I into a product of positive integer as

$$I = \prod_{i=1}^{N} I_i$$

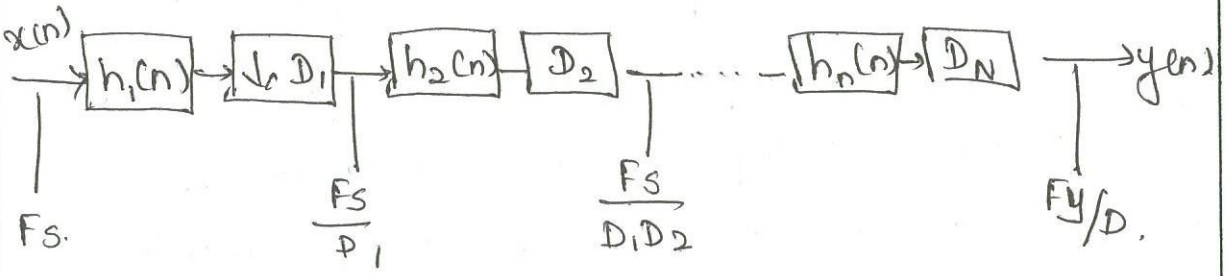
Then each interpolator I_i is implemented and cascaded to get N stages of interpolation and filtering as shown in figure.



Similarly if the decimation factor $D >> 1$ then We express D into productive integer as

$$D = \prod_{i=1}^{N} D_i$$

Each decimator D_i is implemented and cascaded to get N stages of filtering and decimator as shown.



2(ii) Explain Sampling Rate Conversion By a Rational factor I/D. June 13, Dec 12

* In Decimation and Interpolation, the sampling rate conversion is achieved by Integer factor (D and I).

* When sampling rate conversion requires by non-integer factor, it is possible to perform sampling rate conversion by a rational factor I/D.

Steps:

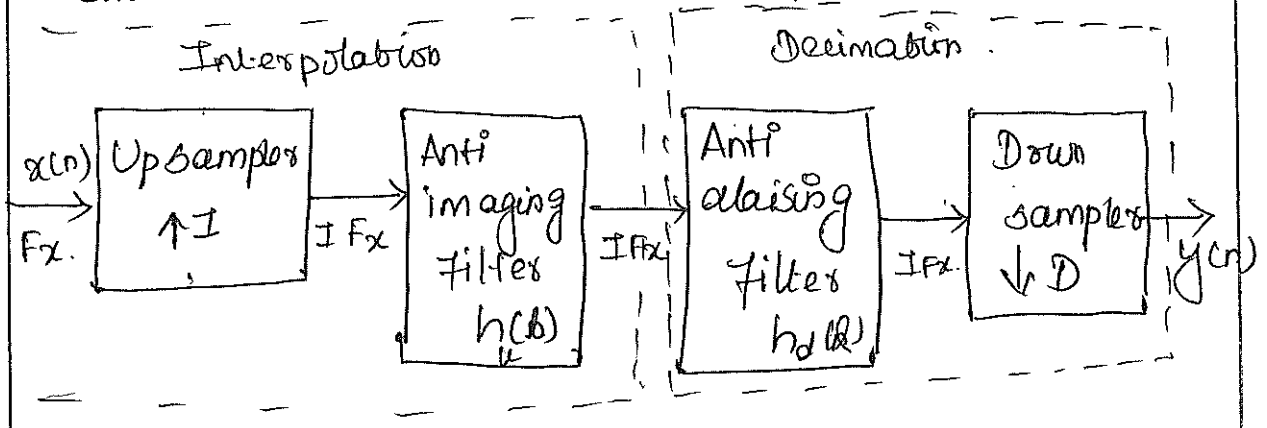
* Perform interpolation by a factor I.
* Filter the output of interpolator using a lowpass filter (anti-imaging filter) with bandwidth $\frac{\Pi}{I}$.
* The output of anti imaging filter is passed through another lowpass filter (anti-aliasing filter) to limit the bandwidth of the signal $\frac{\Pi}{D}$.
* finally the signal band limited to Π/D is Decimated by a factor D.

PRINCIPLE OF CONVERSION BY I/D.

✓ The interpolation by a factor I is obtained first to increase the sampling rate to IFx.

✓ The output of Interpolation is decimated by a factor D, so that the final output rate is $Fy = \frac{I}{D} Fx$.

✓ Such operation can be implemented by cascade connection of interpolation and decimation.



✓ Figure shows there is a cascade of 2 lowpass filters.

✓ The overall cut-off frequency will be minimum of the 2 cut-off frequencies.

✓ The frequency response of anti-imaging filter is given as,

$$H_u(\omega) = \begin{cases} C & ; \frac{-\pi}{I} \le \omega \le \frac{\pi}{I} \\ 0 & 0.W. \end{cases}$$

$$= \begin{cases} I & ; -\pi/I \le \omega \le \frac{\pi}{I} \quad \text{since } C = I. \\ 0 & ; 0.W. \end{cases}$$

✓ The frequency response of anti aliasing filter is given as,

$$H_d(\omega) = \begin{cases} 1 & ; -\pi/D \le \omega \le \pi/D \\ 0 & \text{otherwise} \end{cases}$$

✓ The overall cascade effect of lowpass filters will have cut-off frequency which is minimum

of $\frac{\Pi}{I}$ and $\frac{\Pi}{D}$. Here the frequency response of combined filter is given by

$$H(\omega) = \begin{cases} I & ; |\omega| \le \min\left(\frac{\Pi}{D}, \frac{\Pi}{I}\right) \\ 0 & ; \text{o.w} \end{cases}$$

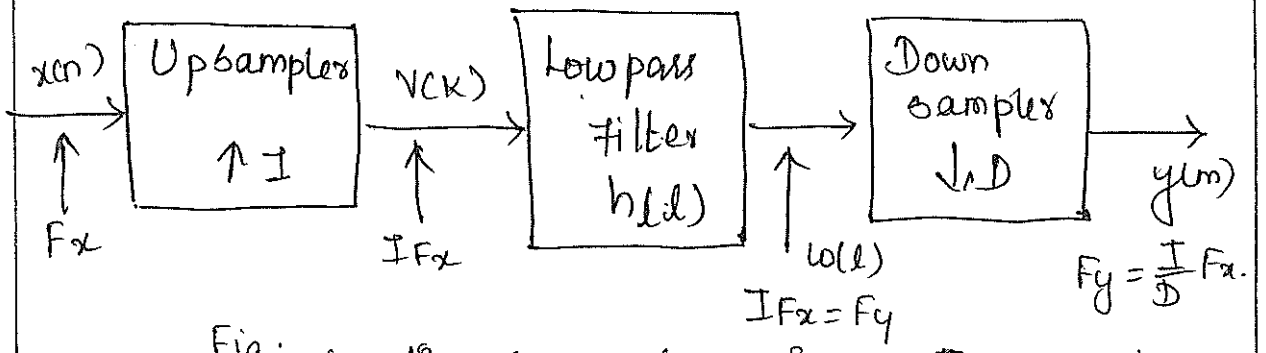


Fig: Sampling rate Conversion by Fraction I/D.

Derivation for Output y(n).

From Figure the output of LPF is given as,

$$\omega(l) = \sum_{k=-\infty}^{\infty} h(l-k)\, V(k).$$

$$\omega(l) = \sum_{k=-\infty}^{\infty} h(l-kI)\, x(k).$$

The output of down sampler is given as,

$$y(m) = \omega(mD)$$

$$y(m) = \sum_{k=-\infty}^{\infty} h(mD - kI)\, x(k).$$

This is the equation for output sequence.

$$Y(e^{j\omega}) = \begin{cases} I/D\ X(e^{j\omega I/D}) & ; \omega = 0 \text{ to } \omega_y \\ 0 & ; \text{otherwise.} \end{cases}$$

$Y(e^{j\omega})$ → Spectrum of sampling rate conversion

3. Explain About- Adpative filters in detail.

* Filtering is the process of removing unwanted noise, and linear filtering is required in various applications.
* Generally a filter will be optimal only if it is designed with some knowledge about the input data.
* If this information is not known the adaptive filters are used.
* The adjustable parameters in the filters are assigned with values based on the estimated statistical nature of the signals.
* So adpative filters are adpatable to the changing environment.

Applications Of Adpative Filter

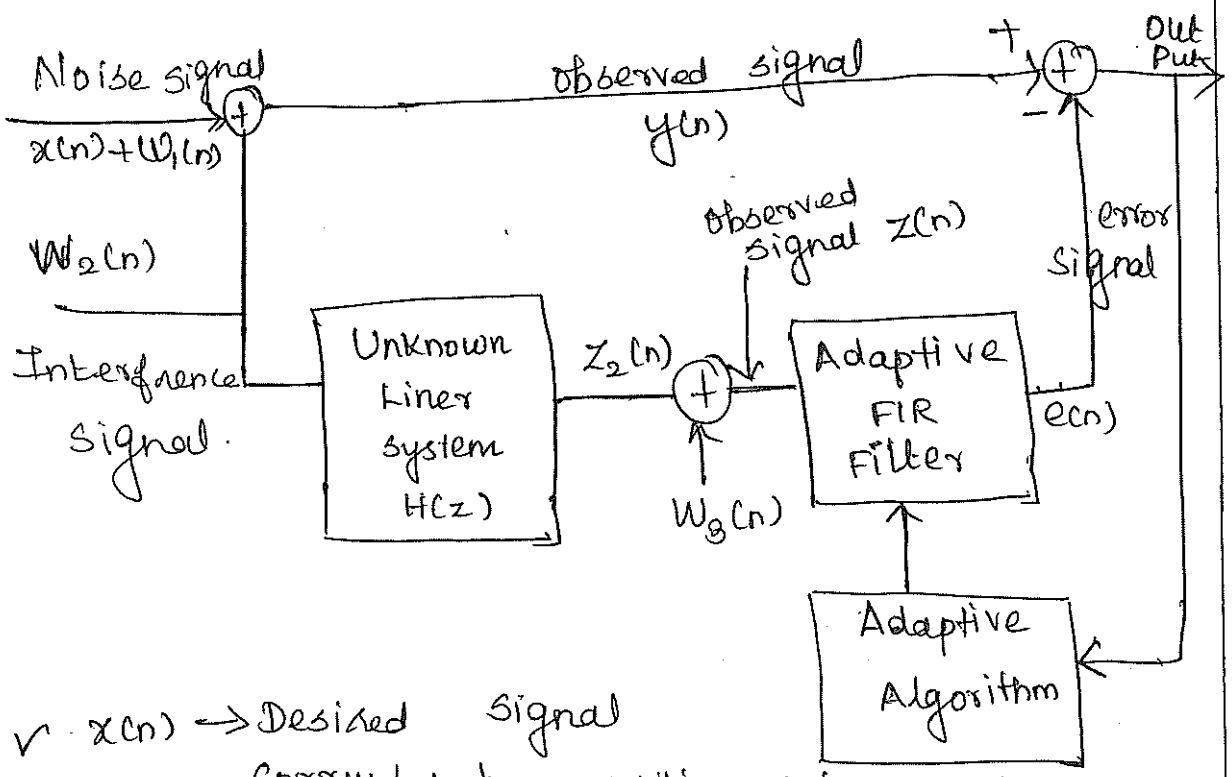
* Adpative Noise Cancelling
* Line enhancing
* Frequency Tracking
* Channel equalisation etc.

Examples of Adpative Filtering

* IIR and FIR filters can used in Adpative Filtering. Mostly FIR filters are used. beause,

* It is simple
* It have only adjustable zeros

Adaptive Noise Cancelling :-



✓ $x(n)$ → Desired signal corrupted by additive noise $W_1(n)$ and interference signal $W_2(n)$.

✓ Observed signal

$$z(n) = Z_2(n) + W_3(n)$$

$Z_2(n)$ → Output of unknown linear system

$W_3(n)$ → Interference Noise.

✓ The estimate of the interference signal $W_2(n)$ is obtained from an FIR filter. The estimate of the desired signal $x(n)$ is the error signal given by

$$e(n) = y(n) - \hat{W}_2(n)$$

$$= y(n) - \sum_{k=0}^{M-1} h(k)\, x(n-k).$$

$\hat{W_2}(n)$ ⟶ is the estimate of $W_2(n)$.

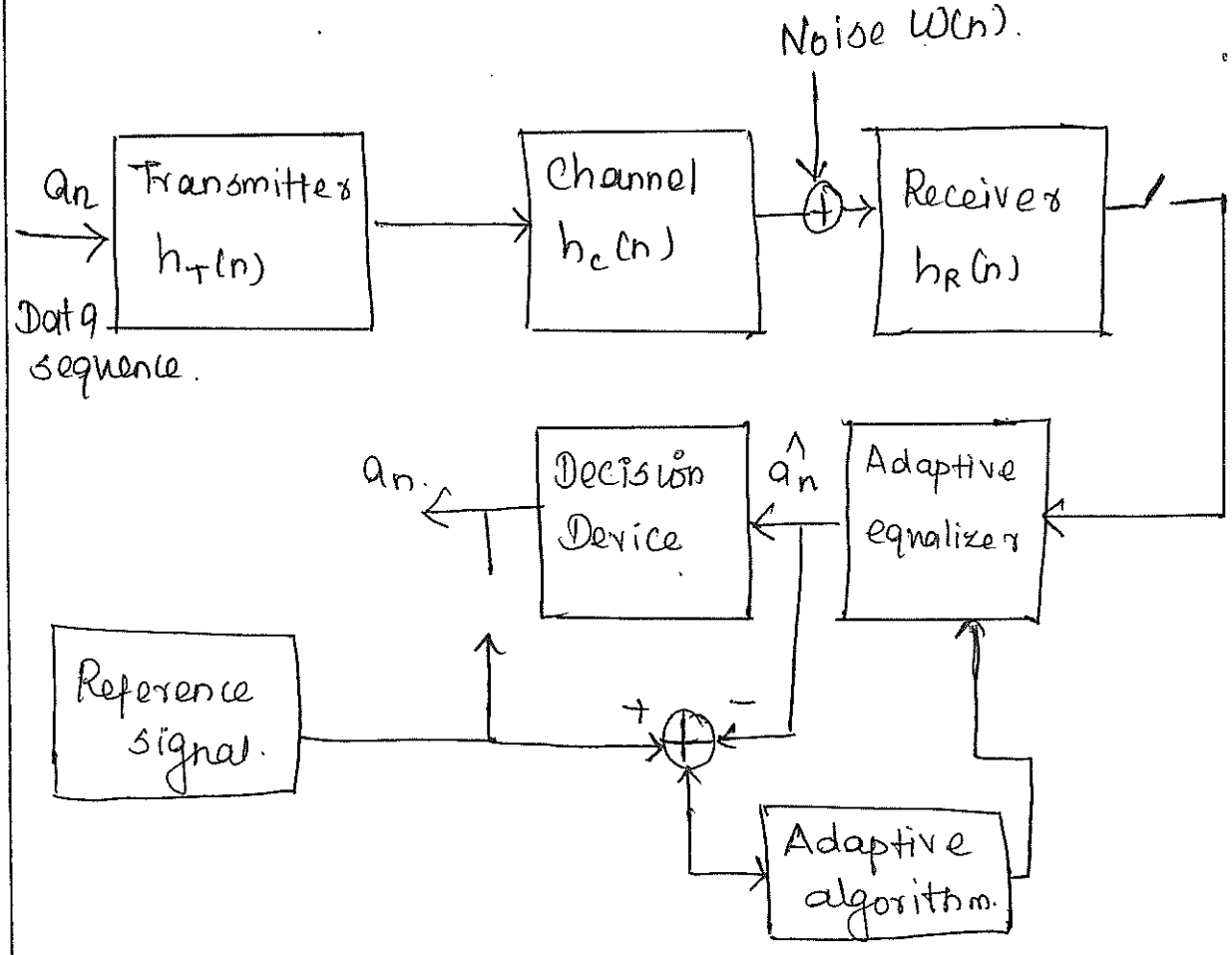
✓ The error signal is used to adjust the FIR filter coefficients.

Adpative Equalisation

Adpative equalization process is done by 2 Methods.

✓ Training Mode

✓ Tracking Mode.



P

- ✓ In training mode, a known test signal is transmitted using a synchronised version of the test signal at the receiver side and comparing this signal with the receiving signal.
- ✓ After comparing these 2 signals, the resultant error signal given the information about the channel.
- ✓ After the training process the adaptive quantiser can be continuously adjusted in the decision directed mode. The error signal is got from the final receiver estimate.
- ✓ The output of the adaptive equalisation is send to the decision device receiver to get the estimate.
- ✓ The error signal estimate is used to adjust the co-efficient of the adaptive equalizer.
- ✓ Adaptive equaliser is mainly used for compensating the channel distortion so that the detected signal will be reliable.

Output Of Receiver Filter.

$$s(t) = \sum_{k=1} a_k P(t - kT_s)$$

$T_s \rightarrow$ Duration of the signalling interval

$P(t) \rightarrow$ Impulse response Of the Cascade Connection

$a_k \rightarrow$ Orginal data Sequence.

Sampled Version Of $s(t)$.

$$S(t) = \sum_{n} a_n P(k-n).$$

$$s(k) = a_k P(0) + \sum_{\substack{n \\ n \neq k}} P(k-n).$$

4. Explain polyphase structure... of Decimator and Interpolator and Polyphase Decomposition Dec 13.

polyphase Decomposition Of FIR filter.

Consider the Transfer function of the FIR filter

$$H(z) = \sum_{n=0}^{N-1} h(0) z^{-n}$$

$$= \sum_{n=0}^{N-1} z^{-n} h(n).$$

On expanding the equation

$$H(z) = z^0 h(0) + z^{-1} h(1) + z^{-2} h(2) + \cdots \quad \text{---(1)}$$

Let us rearrange the above equation in to two section

$$H(z) = \left[z^0 h(0) + z^{-2} h(2) + z^{-4} h(4) + \cdots \right] + \left[z^{-1} h(1) + z^{-3} h(3) + \cdots \right]$$

$$= \left[z^0 h(0) + z^{-2} h(2) + z^{-4} h(4) + \cdots \right] + z^{-1} \left[z^0 h(1) + z^{-2} h(3) + z^{-4} h(5) + \cdots \right].$$

$$= \left[(z^2)^0 h(0) + (z^2)^{-1} h(2) + (z^2)^{-2} h(4) + \cdots \right] + z^{-1} \left[(z^2)^0 h(1) + (z^2)^{-1} h(3) + \cdots \right]$$

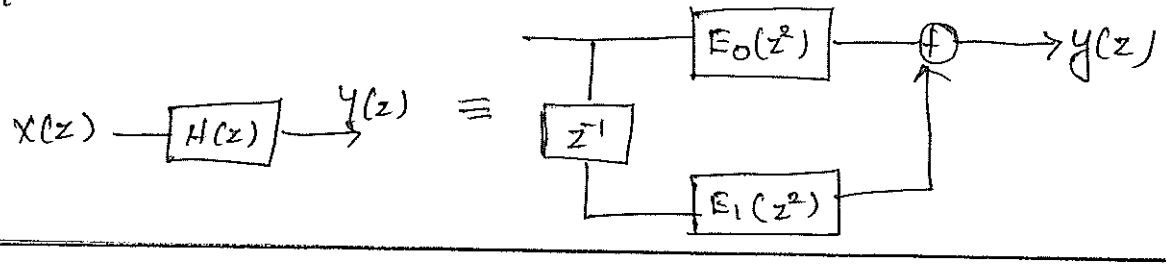
$$\boxed{H(z) = E_0(z^2) + z^{-1} E_1(z^2)}$$

Where $E_0(z^2) = z^0 h(0) + (z^2)^{-1} h(2) + (z^2)^{-2} h(4) + \cdots$

$E_1(z^2) = (z^2)^0 h(1) + (z^2)^{-1} h(3) + (z^2)^{-2} h(5) + \cdots$

here $H(z)$ is decomposed into two section (or) two sub filters.

The polyphase realization of the above equation can be given below.



Let us rearrange the equation (1) in to 3 section

$$H(z) = \left[z^0 h(0) + z^{-3} h(3) + z^{-6} h(6) + \cdots\right] + \left[z^{-1} h(1) + z^{-4} h(4) + z^{-7} (7) + \cdots\right] + \left[z^{-2} h(2) + z^{-5} h(5) + z^{-8} h(8) + \cdots\right]$$

$$= \left(z^0 h(0) + z^{-3} h(3) + z^{-6} h(6)\right) + z^{-1}\left[z^0 h(1) + z^{-3} h(4) + z^{-6} h(7) + \cdots\right] + z^{-2}\left[z^0 h(2) + z^{-3} h(5) + z^{-6} h(8) + \cdots\right]$$

$$= \left[(z^3)^0 h(0) + (z^3)^{-1} h(3) + (z^3)^{-2} h(6) + \cdots\right] + z^{-1}\left[(z^3)^0 h(1) + (z^3)^{-1} h(4) + (z^3)^{-2} h(7) + \cdots\right] + z^{-2}\left[(z^3)^0 h(2) + (z^3)^{-1} h(5) + (z^3)^{-2} h(8) + \cdots\right]$$

$$H(z) = E_0(z^3) + z^{-1} E_1(z^3) + z^{-2} E_2(z^3)$$

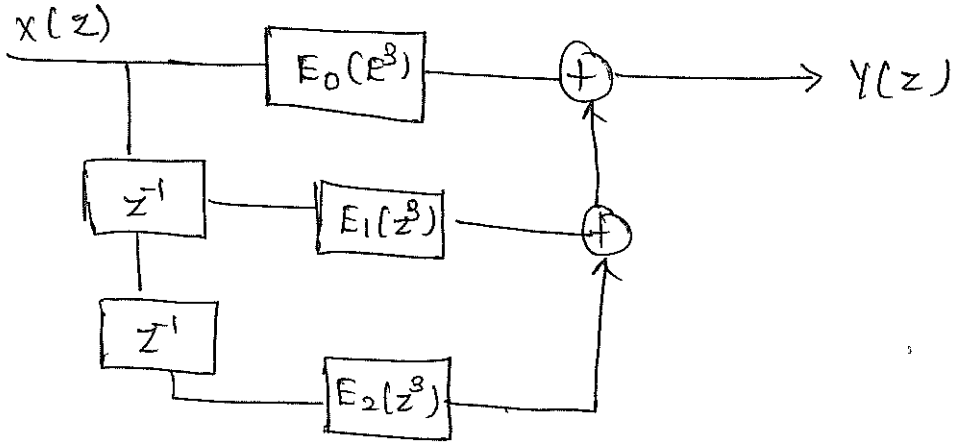
Where

$$E_0(z^3) = (z^3)^0 h(0) + (z^3)^{-1} h(3) + (z^3)^{-2} h(6) + \cdots$$

$$E_1(z^3) = (z^3)^0 h(1) + (z^3)^{-1} h(4) + (z^3)^{-2} h(7) + \cdots$$

$$E_2(z^3) = (z^3)^0 h(2) + (z^3)^{-1} h(5) + (z^3)^{-2} h(8) + \cdots$$

The polyphase realizabion is



In general the polyphase decomopositibn into L seebion

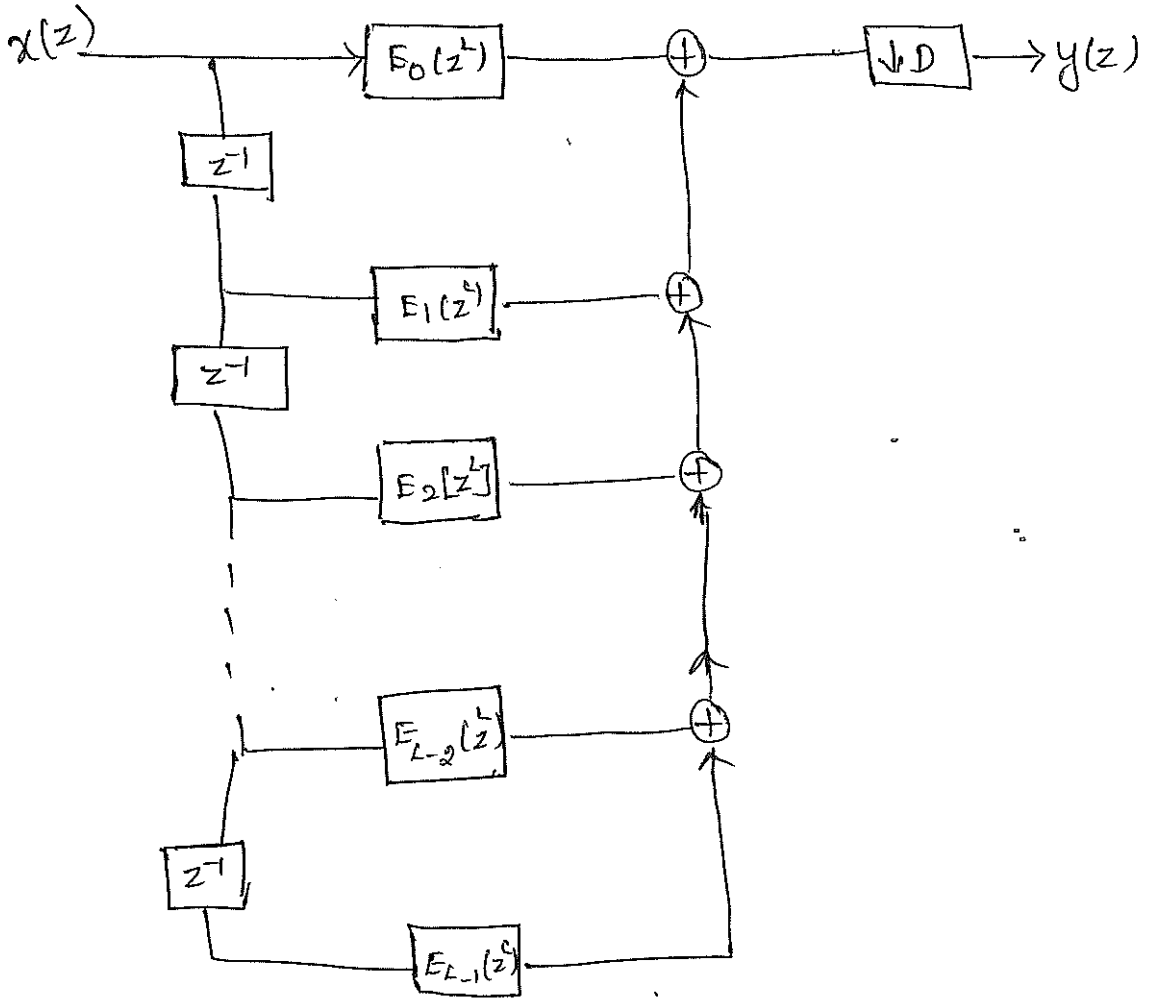
$$H(z) = \sum_{m=0}^{L-1} z^{-m} E_m(z^L).$$

Polyphase Structure Of Decimator

In decimator a lowpass filter called antialiasing filter is employed at the input In order to band limit the input signal, so that aliasing is avoided in the output spectrum decimator.

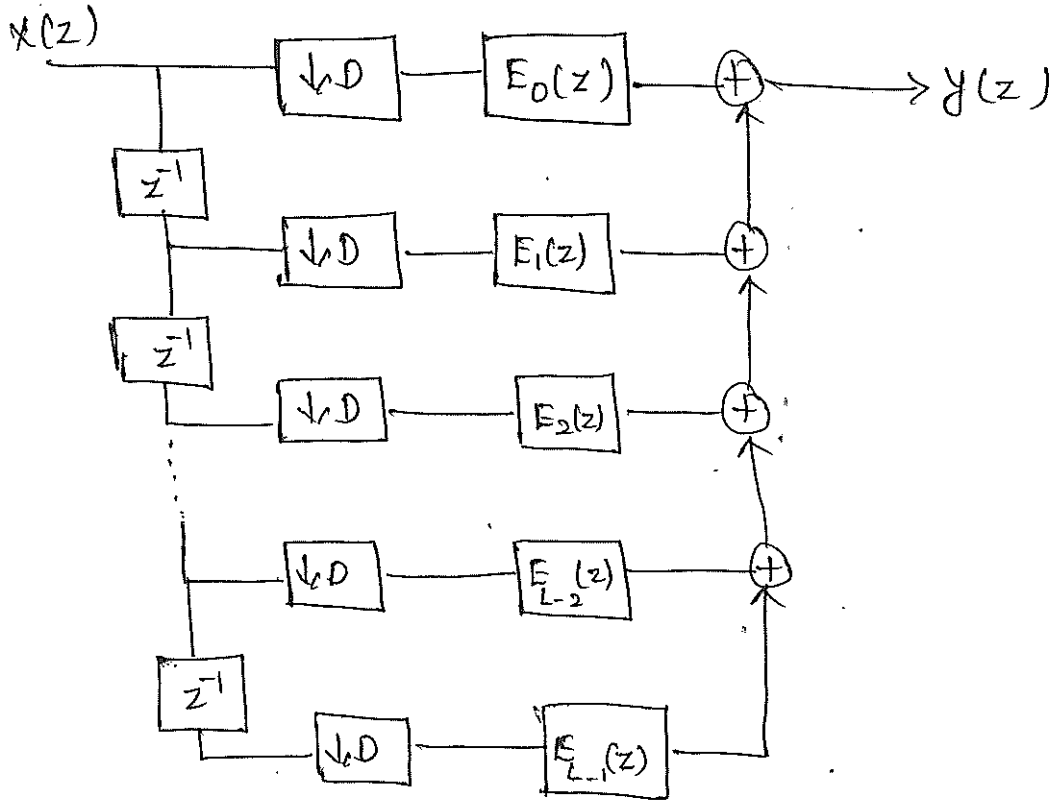
Consider a decimator with sampling rate reduction factor D. Let H(z) be the transfer function of low pass antialiasing FIR filter.

In order to reduce the computations in FIR filter polyphase decomposition can be applied to FIR filter to decompose into L sub-filters as shown in Fig.



Efficient structure:

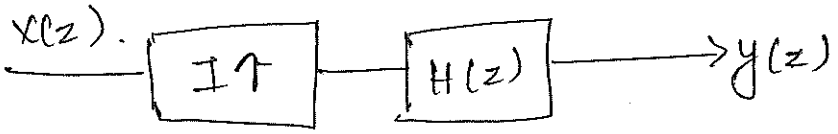
Further reduction in computations can be obtained in decimator is shifted to input of sub filters. as shown in fig.

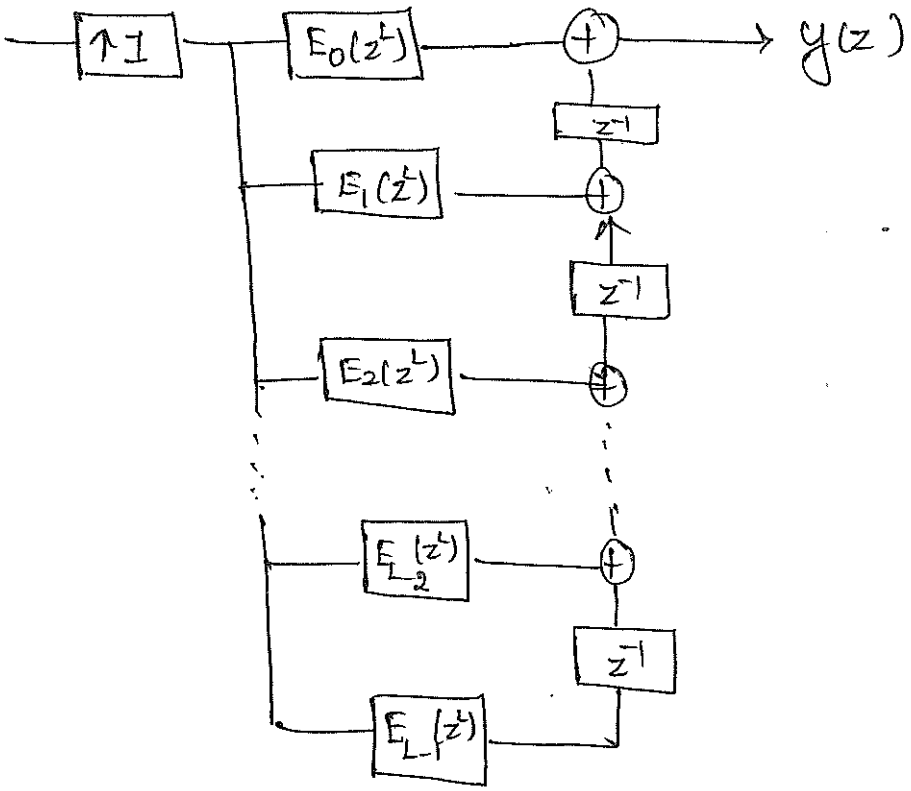


Polyphase structure of Interpolator.

In interpolator a low pass filter called anti-imaging filter is employed at the output in order to eliminate the multiple images in the output spectrum of interpolator.

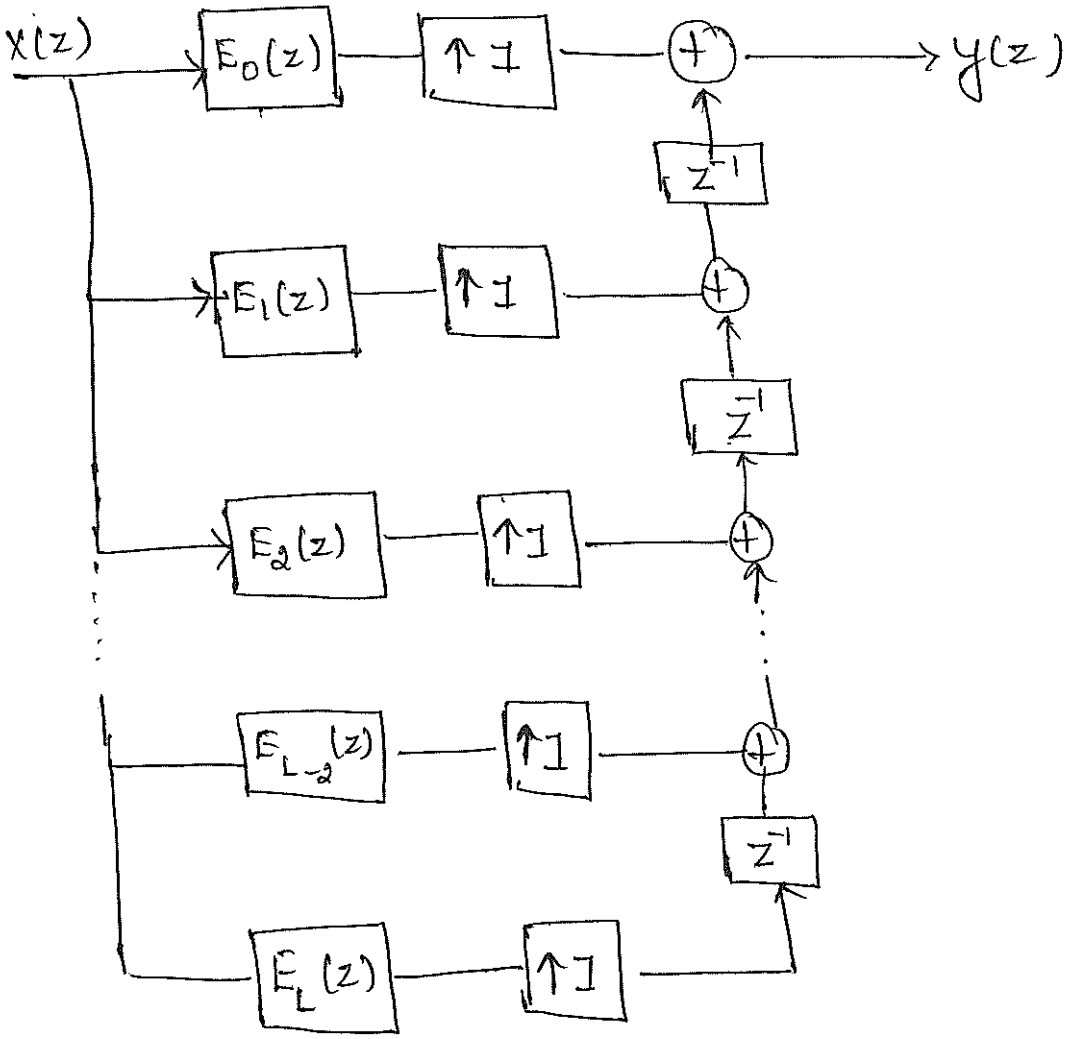
Consider an interpolator with sampling rate multiplication factor 'L'.





The interpolator in the structure will introduce large number of zeros in the input signal. so that only one in every L samples will be non-zero.

Therefore further reduction in computations can be obtained the interpolator is shifted to output of sub filters as shown in figure.



5. The transfer function of an FIR filter is

$$H(z) = 0.2 + 0.7z^{-1} + 0.8z^{-2} + 0.15z^{-3} + 0.6z^{-4} + 0.32z^{-5} + 0.5z^{-6} + 0.4z^{-7} + 0.9z^{-8}.$$

Perform polyphase decomposition of $H(z)$ to decompose into 2 section.

Soln:

Let us express $H(z)$ as 2 section

$$H(z) = [0.2 + 0.8z^{-2} + 0.6z^{-4} + 0.5z^{-6} + 0.9z^{-8}] + [0.7z^{-1} + 0.15z^{-3} + 0.32z^{-5} + 0.4z^{-7}]$$

$$= \left[0.2 + 0.8z^{-2} + 0.6z^{-4} + 0.5z^{-6} + 0.9z^{8}\right] + z^{-1}\left[0.7 + 0.15z^{-2} + 0.32z^{-4} + 0.4z^{-6}\right]$$

$$= \left[(z^2)^2 0.2 + 0.8(z^2)^{-1} + 0.6(z^2)^{-2} + 0.5(z^2)^{-3} + 0.9(z^2)^{-4}\right] + z^{-1}\left[0.7(z^2)^0 + 0.15(z^2)^{-1} + 0.32(z^2)^{-2} + 0.4(z^2)^{-3}\right]$$

$$\boxed{H(z) = E_0(z^2) + z^{-1}E_1(z^2)}$$

6. The transfer function of an IIR filter is $H(z) = \frac{1+0.7z^{-1}}{1-0.9z^{-1}}$. Perform polyphase decomposition of $H(z)$ to decompose into 2 section.

Soln:

$$H(z) = \frac{1+0.7z^{-1}}{1-0.9z^{-1}}.$$

Let choose polynomial $P(z) = 1+0.9z^{-1}$

Let us multiply the numarator and denominator of $H(z)$ by $P(z)$

$$H(z) = \frac{1+0.7z^{-1}}{1-0.9z^{-1}} \times \frac{1+0.9z^{-1}}{1+0.9z^{-1}}$$

$$= \frac{1+0.9z^{-1}+0.7z^{-1}+0.63z^{-2}}{1-0.81z^{-2}}.$$

$$H(z) = \frac{1 + 1.6z^{-1} + 0.63z^{-2}}{1 - 0.81z^{-2}}$$

$$= \frac{1 + 0.63z^{-2}}{1 - 0.81z^{-2}} + \frac{1.16z^{-1}}{1 - 0.81z^{-2}}$$

$$= \frac{1 + 0.63(z^2)^{-1}}{1 - 0.81(z^2)^{-1}} + \frac{z^{-1}\ 1.16}{1 - 0.81(z^2)^{-1}}$$

$$H(z) = E_0(z^2) + z^{-1} E_1(z^2)$$

Where

$$E_0(z^2) = \frac{1 + 0.63(z^2)^{-1}}{1 - 0.81(z^2)^{-1}}$$

$$E_1(z^2) = \frac{1.16}{1 - 0.81(z^2)^{-1}}$$

Show that the upsampler and downsampler are time varient system.

Solution

Consider a factor of L upsampler defined by

$$y(n) = x\left(\frac{n}{L}\right)$$

The output due to delayed input is

$$y(n,k) = x\left(\frac{n}{L} - k\right)$$

The delayed output is

$$y(n-k) = x\left(\frac{n-k}{L}\right)$$

$$y(n,k) \neq y(n-k)$$

Therefore the upsampler is a time varient system. similarly for down sampler

$$y(n) = x(nM)$$

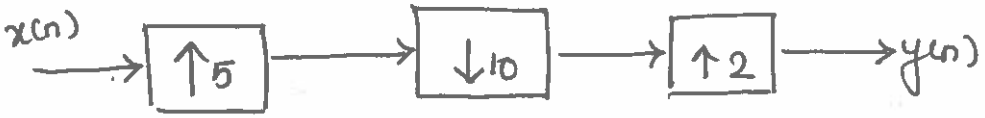
$$y(n,k) = x(nM - k)$$

$$y(n-k) = x(M(n-k))$$

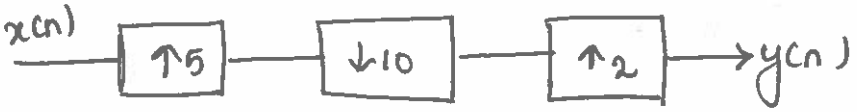
$$y(n,k) \neq y(n-k)$$

Therefore the downsampler is time varient system.

For the multirate systems shown in figure develop an expression for the output $y(n)$ as a function of the input $x(n)$



Solution



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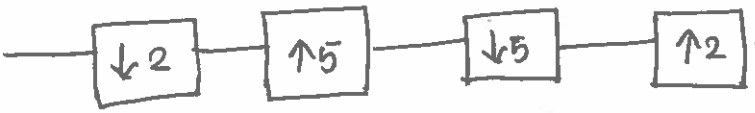

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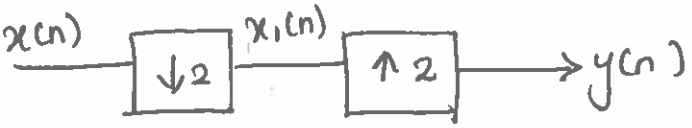


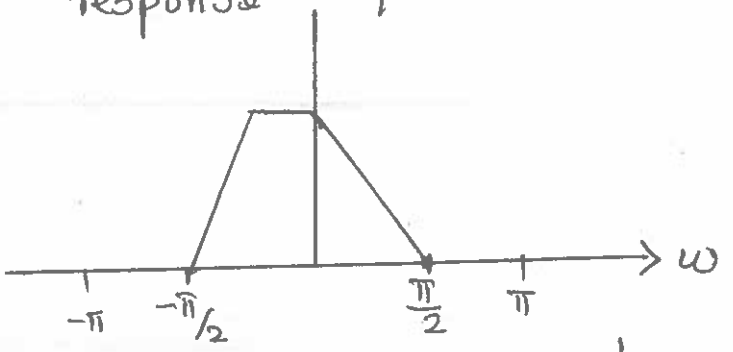
$\therefore \quad x_1(n) = x(2n)$ and $y(n) = x_1\left(\frac{n}{2}\right)$ for $n = 2k$

$= 0$ otherwise

$\Rightarrow y(n) = x(n)$ for $n = k$

$= 0$ otherwise.

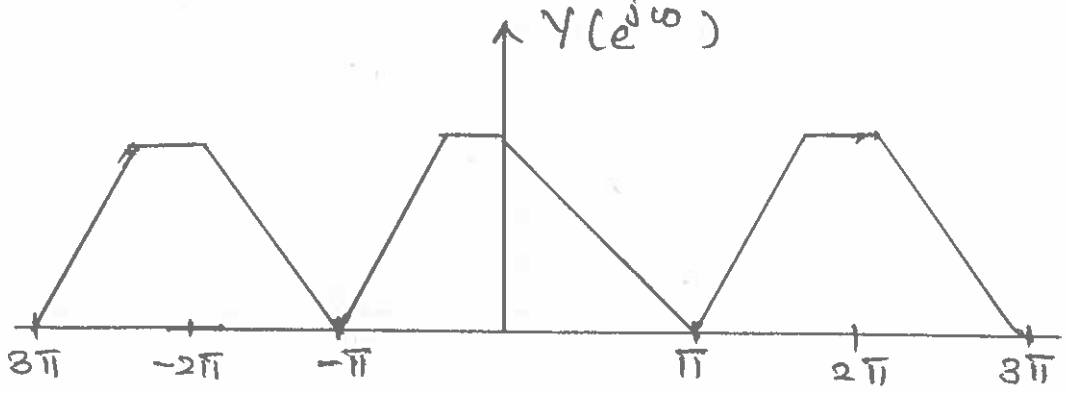
The frequency response of x(n) is shown in fig.



If the input is passed through a down sampler by 2, Find the frequency response of output and give your comment on aliasing.

Solution

$$Y(e^{j\omega}) = \frac{1}{2}\left[x(e^{j\omega/2}) + x(e^{-j\omega/2})\right]$$



The frequency response $Y(e^{j\omega})$ is shown in fig We can find that there is no aliasing.

Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 71735

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2017.

Fifth /Sixth Semester

Electronics and Communication Engineering

EC 6502 — PRINCIPLES OF DIGITAL SIGNAL PROCESSING

(Common to Biomedical Engineering, Medical Electronics)

(Regulations 2013)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. What is the relation between DTFT and DFT?

2. Compute the DFT of the sequence $x(n) = \{1, -1, 1, -1\}$.

3. What are the requirements for the digital filter to be stable and casual?

4. Discuss the need for prewarping.

5. What is Gibbs phenomenon?

6. Compare Hamming window with Blackmann window.

7. What are the methods used to prevent overflow?

8. What is meant by "dead band" of the filter?

9. Define adaptive filtering.

10. List the applications of multirate signal processing.

PART B — (5 × 16 = 80 marks)

11. (a) Compute the DFT for the sequence $\{1, 2, 3, 4, 4, 3, 2, 1\}$. Using radix – 2 DIF – FFT algorithm. (16)

Or

(b) In an LTI system the input $x(n) = \{1, 1, 2, 1\}$ and the impulse response $h(n) = \{1, 2, 3, 4\}$. Perform the circular convolution using DFT and IDFT. (16)

12. (a) Design a digital Butterworth filter with the following specifications

$0.707 \le |H(e^{jw})| \le 1,\ 0 \le \omega \le 0.5\pi$

$|H(e^{jw})| \le 0.2,\ 0.75\pi \le \omega \le \pi$

Determine system function $H(z)$ for a Butterworth filter using Bilinear transformation. (16)

Or

(b) Determine the system function of the lowest order digital Chebyshev filter with the following specifications, 3db ripple in the pass band $0 \le \omega \le 0.2\pi$ and 25db attenuation in the stop band $0.45\ \pi \le \omega \le \pi$. (16)

13. (a) Design a HPF with the following frequency response

$H_d(e^{jw}) = 1$ for $\pi/4 \le |\omega| \le \pi$

$= 0$ for $|\omega| \le \pi/4$

of length $N = 11$ using Hanning window. (16)

Or

(b) Determine the coefficients of a linear phase FIR filter of length N = 15 which has a symmetric unit sample response and a frequency response that satisfies the conditions. (16)

$H(2\pi k/15) = 1$; *for* $k = 0, 1, 2, 3$

$= 0$; *for* $k = 4, 5, 6, 7$

14. (a) Two first order filters are connected in cascaded whose system functions of the individual sections are $H1(z) = 1/(1 - 0.5z^{-1})$ and $H2(z) = 1/(1 - 0.6z^{-1})$. Determine the overall output noise power. (16)

Or

(b) Explain the characteristics of limit cycle oscillations with respect to the system described by the difference equation $y(n) = 0.95y(n-1) + x(n)$. Determine the dead band. (16)

15. (a) Explain the concept of deciation by a factor D and interpolation by factor I. With help of equation explain sampling rate conversion by a rational factor I/D. (16)

Or

(b) Explain the operation of adaptive filter with suitable diagrams and equations. (16)

Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 80343

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2016.

Fifth Semester

Medical Electronics

EC 6502 — PRINCIPLES OF DIGITAL SIGNAL PROCESSING

(Common to Electronics and Communication Engineering and Sixth Semester Biomedical Engineering)

(Regulations 2013)

www.recentquestion paper.com

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Compare Radix 2 DIT, DIF FFT algorithm.

2. Test the causality and stability of $y(n) = \sin x(n)$.

3. What is known as prewarping?

4. What are the properties of bilinear transformation?

5. What do you understand by linear phase response?

6. What are the desirable characteristics of the window?

7. What are the different types of fixed point representation?

8. Name the three quantization error due to finite word length registers in digital filters.

www.recentquestion paper.com

9. What is the need for anti imaging filter after upsampling signal?

10. What is meant by adaptive filter?

PART B — (5 × 16 = 80 marks)

11. (a) Derive radix 2 – DIT FFT algorithm and obtain DFT of the sequence $x(n) = \{1,2,3,4,4,3,2,1\}$ using DIT algorithm. (16)

Or

www.recentquestion paper.com

(b) (i) Compute IDFT of the sequence $X(K) = \{7, -0.707, -j0.707, -j, 0.707 - j0.707, 1, 0.707 + j0.707\ j, -0.707 + j\ 0.707\}$ using DIF algorithm. (10)

(ii) Perform the linear convolution of finite duration sequences $h(n) = \{1,2\}$ and $x(n) = \{1,2,-1,2,3,-2,-3,-1,1,2,-1\}$ by overlap save method. (6)

12. (a) Design a third order Butterworth digital filter using impulse invariant technique. Assume sampling period T = 1 sec. (16)

Or

(b) Convert the single pole low pass filter with system function $H(z) = \frac{0.5(1+z^{-1})}{1-0.302z^{-2}}$ into band pass filter with upper and lower cut off frequencies w_u & w_L respectively. The LPF has 3dB BW of $w_p = \frac{\pi}{6}$ & $w_u = \frac{3\pi}{4}$, $w_l = \frac{\pi}{4}$.

www.recentquestion paper.com

13. (a) Design an ideal BPF with a frequency response

$$H\alpha(e^{jw}) = \begin{cases} 1, & for\ \frac{\pi}{4} \le |w| \le \frac{3\pi}{4} \\ 0, & otherwise \end{cases}$$

Find the value of $h(n)$ for $N = 11$ and plot the frequency response. (16)

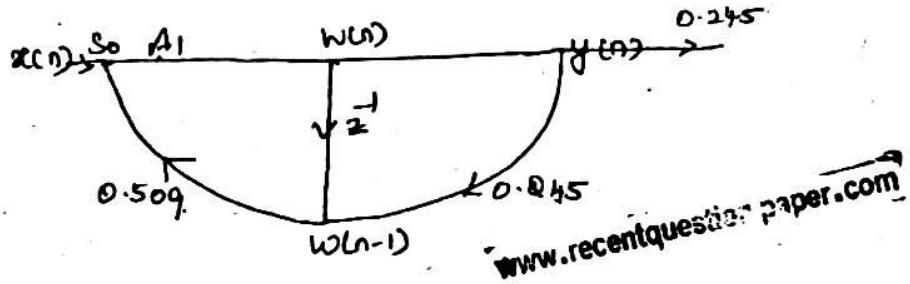
Or

(b) Design a linear phase FIR filter with a cut off frequency of $\frac{\pi}{2}$ r/sec. Take $N = 17$ using frequency sampling techniques. (16)

14. (a) Study the limit cycle behaviour of the system described by $y(n) = Q[a\,y(n-1)] + x(n)$, where $y(n)$ is the output of the filter and $Q[.]$ is quantization. Assume $a = \frac{7}{8}$, $x(0) = \frac{3}{4}$ & $x = 0$, for $n > 0$ choose 4 bit sign magnitude. (16)

Or

(b) For the digital network shown in figure find $H(z)$ and scale factor. So to avoid over flow register A_1 (16)



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15. (a) (i) Explain in about detail the multistage implementation of sampling rate conversion. (8)

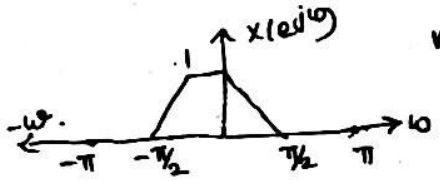
(ii) For the multirate system shown in figure develop an expression for the output $y(n)$ as a function of i/p $x(n)$ (8)



Or

(b) (i) Show that the upsampler and down sampler are time variant systems. (8)

(ii) The frequency response of $x(n)$ is shown in figure



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If the input is passed through a down sampler by 2, find the frequency response of output and give your comment on aliasing. (8)

www.recentquestion paper.com

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2015.

Fifth Semester.

Electronics and Communication Engineering.

EC 2302/EC 52 - DIGITAL SIGNAL PROCESSING.

(Regulation 2008).

Time: Three hours Maximum: 100 marks

PART - A (10 x 2 = 20 marks).

1. Write the Analysis and Synthesis equations of DFT. [Page 13]

2. Is the DFT of a finite length sequence periodic? If so, State the theorem? [Page: 12]

3. Find H(z) for the IIR filter whose $H(s) = \frac{1}{(S+6)}$ with T = 0.1 sec. [Page: 48]

4. Draw the response curve for butterworth, Chebychev and elliptic filters. [Page: 38]

5. Why is window function used in FIR filter design? [Page: 72]

6. Draw a Causal FIR filter structure for length m = 5.

7. Explain briefly quantization noise.

8. List the types of limit cycle Oscillation.

9. State the basic Operations of multirate Signal processing. [Page: 116]

10. What is pipelining?

PART- B (5 × 16 = 80 marks).

11.(a) (i) Determine the 8-point DFT of the sequence $x(n) = \{0, 0, 1, 1, 1, 0, 0, 0\}$ (8) [Page: 16]

(ii) What is decimation in frequency algorithm Write the similarities and differences between DIF and DIT algorithms (8) [Page: 14]

Or

(b) With appropriate diagrams discuss how Overlap – Save method and Overlap add method are used. [Page: 30] (16)

12. (a) Explain in detail the steps involved in the design of IIR filter using bilinear transformation. [Page: 49] (16)

Or

(b) (i) Convert the Analog filter with system function $H_a(s) = \dfrac{s+0.1}{(s+0.1)^2+16}$ into a digital IIR filter by means of the bilinear transformation The digital filter is to have a resonant frequency of $\omega_r = \frac{\pi}{2}$. [Page: 48] (10)

(ii) Draw the structure for the IIR filter in direct form – II for the following transfer

function.

$$H(z) = \frac{(2+3z^{-1})(4+2z^{-1}+3z^{-2})}{(1+0.6z^{-1})(1+z^{-1}+0.5z^{-2})} \quad (6)$$

[Page: 40]

13. (a) Explain the principle and procedure for designing FIR filter using rectangular window. [Page 78] (16)

Or

(b) Design a FIR linear phase digital filter for the response. [Page: 84]

$$|H(\omega)| = \begin{cases} 1 & \text{for } |\omega| \le \frac{\pi}{6} \\ 0 & \text{for } \frac{\pi}{6} < |\omega| \le \pi \end{cases} \quad (16)$$

14. (a) (i) Explain the various formats of the fixed point representation of binary numbers. [Page: 98] (8)

(ii) What is meant by finite word length effects on digital filters? List them. (8)

Or

(b) Find the Output round off noise power for the ~~System~~ having transfer function $H(z) = \frac{1}{(1-0.5z^{-1})(1+0.4z^{-1})}$ which is realized in Cascade form. Assume word length is 4 bits. [Page: 109] (16)

15. (a) Explain in detail the two basic Operations in multirate Signal processing. (16)

[Page: 120]

Or

(b) (i) Draw and Explain the block diagram of Subband Coding System. (8)

(ii) Discuss About the musical Sound processing. (8)

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B.E. / B.Tech. DEGREE EXAMINATION,
MAY / JUNE 2014.

Fifth Semester

Electronics And Communication Engineering

EC 2302 – DIGITAL SIGNAL PROCESSING

(Regulation 2008)

Time: Three hours Maximum: 100 marks

PART A – (10 X 2 = 20 marks)

1. State the difference between DFT and DTFT. [Page: 11]
2. What is Bit Reversal? [Page: 11]
3. Sketch the frequency response of an odd and an even Order Chebyshev low pass filters. [Page: 38]
4. What is bilinear transformation? What is the main Advantages and disadvantages of this technique? [Page: 37]
5. State the effect of having abrupt discontinuity in frequency response of FIR filters. [Page: 72]
6. State Gibb's phenominon. [Page: 72]
7. State the need for Scaling in filter implementation. [Page: 114]

8. What is product round off noise? [Page:94]

9. Write the need for decimation. [Page:116]

10. What is meant by multirate Signal processing? [Page:116]

PART B - (5 x 16 = 80 marks).

11. (a) Explain in detail about Overlap Add method and Overlap Save method for filtering of long data Sequences Using DFT [Page:30] (16)

Or

(b) Develop a 8 point DITFFT algorithm. Draw the Signal flow graph. Determine the DFT of the following Sequence. $x(n) = \{1, 1, 1, 1, 0, 0, 0, 0\}$ Using the Signal flow graph. Show all the intermediate results on the Signal flow graph. [Page: 26] (16)

12. (a) Design a low pass Butterworth digital filter with the following Specifications, $W_s = 4000$, $W_p = 3000$ $A_p = 3$ dB, $A_s = 20$ dB $T = 0.0001$ Sec [Page: 49] (16)

Or

(b) A System is represented by a transfer function $H(z)$ is given by

$$H(z) = 3 + \frac{4z}{z-1/2} - \frac{z}{z-1/4}.$$

(i) Does this $H(z)$ represent a FIR or IIR filter Why? [Page: 40] (4)

(ii) Give a difference Equation realization of this System using direct form-I (6)

(iii) Draw the block diagram for the direct form II Canonic realization, and give the governing Equations for implementation (6)

13. (a) Explain the designing of FIR filters using frequency Sampling method. (16) [Page: 90]

Or.

(b) (i) State and Explain the properties of FIR filters. State their importance. (8) [Page: 71]

(ii) Explain linear phase FIR Structures. What are the Advantages of Such Structures? (8)

14. (a) (i) Explain the Characteristics of limit Cycle Oscillation with respect to

The System described by the difference equation:

$y(n) = 0.95\, y(n-1) + x(n)$;

$x(n) = 0$ and $y(-1) = 13$.

Determine the dead band range of the System [Page: 107] (10)

(ii) Explain the effects of Co-efficient Quantization in FIR filters. [Page: 113] (6)

Or

(b) (i) Derive the Signal to Quantization noise ratio of A/D Converter. [Page: 101] (6)

(ii) Compare the truncation and rounding Errors using fixed point and floating point representation. (10)

15. (a) Explain in detail about the polyphase implementation of FIR filters for interpolator and decimators. [Page: 133] (16)

Or

(b) (i) Write notes on the following:

(1) Over Sampling A/D Converter.

(2) Over Sampling D/A Converter (10)

(ii) State the Various Applications of multirate Signal processing. [Page: 129] (6)

B.E. / B. Tech. DEGREE EXAMINATION,
NOVEMBER / DECEMBER 2014.

Fifth Semester

Electronics And Communication Engineering

EC 2302 - DIGITAL SIGNAL PROCESSING.

(Regulation 2008)

Time: Three hours Maximum: 100 marks.

PART- A (10 x 2 = 20 marks)

1. Compare the number of multiplications required to compute the DFT of a 64 point sequence using direct computation and that using FFT. [Page: 10]

2. What is meant by 'in place' in DIT and DIF algorithms? [Page: 12]

3. Distinguish between Butterworth and Chebyshev filter. [Page: 36]

4. What is prewarping? [Page: 36]

5. Give the equations specifying Hamming and Blackman Window. [Page: 72]

6. Realize the following Causal linear phase FIR system function

$$H(z) = \frac{2}{3} + z^{-1} + \frac{2}{3} z^{-2}.$$

7. What is Scaling? Page: 114

8. What is dead band of a filter? Page: 95

9. Define decimator and interpolator. Page: 116

10. List the Applications of multirate Signal processing. [Page 119]

PART B - (5 × 16 = 80 marks)

11. (a) (i) Find the DFT of a sequence $x(n) = \{1, 2, 3, 4, 4, 3, 2, 1\}$ using DIT algorithm. [Page: 26] (10)

(ii) State any six properties of DFT. (6)

Or.

(b) (i) Using linear Convolution find $y(n) = x(n) * h(n)$ for the sequences $x(n) = \{1, 2, -1, 2, 3, -2, -3, -1, 1, 1, 1, 2, -1\}$ and $h(n) = \{1, 2\}$ Compare the result by Solving the problem using Overlap add method and Overlap Save method. [Page: 30] (12)

(ii) Find the IDFT of the sequence,

$$X(K) = \{6, -2+2j, -2, -2-2j\}$$

using DIF algorithm. (4)

12. (a) Design a digital Chebyshev filter to satisfy the constraints

$$0.707 \leq |H(e^{j\omega})| \leq 1 \; ; \quad 0 \leq \omega \leq 0.2\pi$$

$$|H(e^{j\omega})| \leq 0.1 \; ; \quad 0.5\pi \leq \omega \leq \pi$$

Using bilinear transformation and assuming $T = 1$ sec.

[Page: 55] (16)

(or)

(b) (i) For the analog transfer function

$$H(s) = \frac{2}{(s+1)(s+2)}$$

Determine $H(z)$ using impulse invariant method. Assume $T = 1$ sec. [Page: 46] (10)

(ii) Obtain the cascade and parallel realization for the system function given by,

[Page: 40]

$$H(z) = \frac{1 + \frac{1}{4}z^{-1}}{\left(1 + \frac{1}{2}z^{-1}\right)\left(1 + \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2}\right)} \quad (6)$$

13. (a) (i) A low pass filter has a desired response as given below,

$$H_d(e^{j\omega}) = \begin{cases} e^{-j3\omega} & ; \quad 0 \leq \omega \leq \frac{\pi}{2} \\ 0 & ; \quad \frac{\pi}{2} \leq \omega \leq \pi. \end{cases}$$

Determine the filter coefficients $h(n)$ for

M=7, using type-I frequency sampling technique. [Page: 90] (10)

Or

(b) Design a bandpass filter which approximates the ideal filter with cut off frequencies at 0.2 rad/sec and 0.3 rad/sec. The filter order is M=7. Use the hanning window function. [Page: 80] (16)

14. (a) Discuss the following: [Page: 109]

(i) Product quantization error. (8)

(ii) Limit cycle oscillations. (8)

[Page: 107]

(Or)

(b) (i) Derive the equation for rounding and truncation error (8)

(ii) Derive the equation for quantization noise power. [Page: 101] (8)

15) (a) Explain with block diagram the general poly phase frame work for decimator and interpolator. (16)

[Page: 133]

Or

(b) Implement a two stage decimator for the following specifications.

Sampling rate of the input signal = 20,000 Hz.

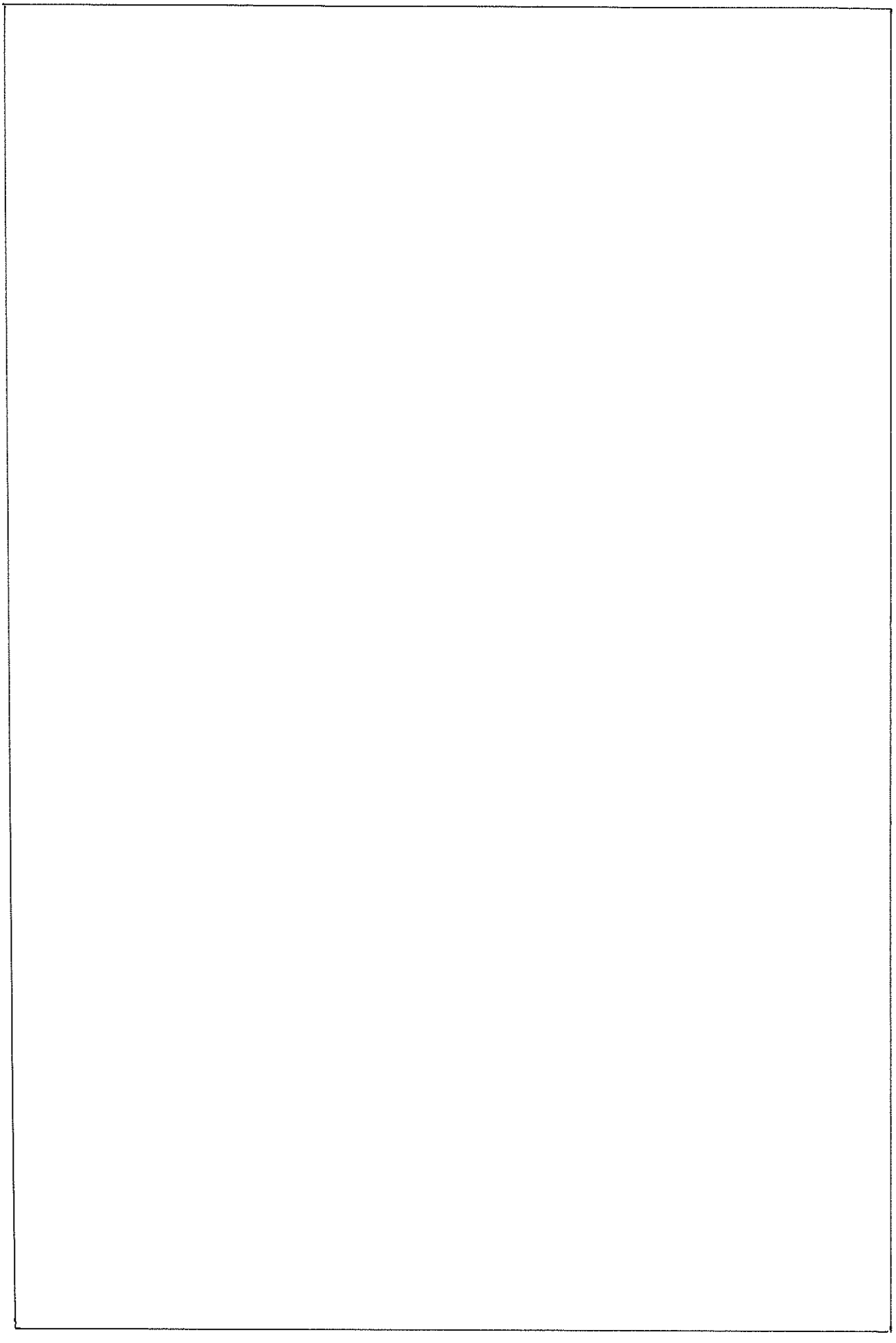
M = 100

Pass band = 0 to 40 Hz.

Transition band = 40 to 50 Hz.

Pass band ripple = 0.01.

Stop band ripple = 0.002. (16)



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B.E. / B.Tech. DEGREE EXAMINATION,

NOVEMBER / DECEMBER 2013.

Fifth Semester.

Electronics and Communication Engineering.

EC 2302 – DIGITAL SIGNAL PROCESSING.

(Regulation 2008)

Time: Three hours. Maximum: 100 marks.

PART A – (10 × 2 = 20 marks)

1. What is Zero padding? What is the purpose of it? [Page: 10]

2. How many multiplications and additions are required to Compute N-point DFT using radix-2 FFT? [Page: 10]

3. Give the steps in the design of a digital filter from Analog filters.

4. What are the disadvantages of direct form realisation? [Page: 39]

5. State the properties of FIR filter. [Page: 71]

6. Give the desirable Characteristics of the Window. [Page: 70]

7. What do you understand by input quantization error? [Page: 94]

8. State the methods used to prevent Overflow. [Page: 97]

9. Give the Steps in multistage Sampling rate Converter design. [Page: 124]

10. Write Any four Applications of multi rate Signal processing. [Page: 119]

PART B – (5 × 16 = 80 marks)

11. (a) (i) Compute the DFT of the Sequence whose Values for One period is given by $x(n) = \{1, 1, -2, -2\}$. [Page: 16] (8)

(ii) Compute the Eight point DFT of the Sequence

$$x(n) = \begin{cases} 1 & 0 \le n \le 7 \\ 0 & \text{Otherwise} \end{cases}$$

by using DIT and DIF algorithms. [Page: 28] (8)

Or

(b) (i) Summarize the difference between Overlap Save method and Overlap add method. [Page: 30] (8)

(ii) Evaluate the 8-point DFT for the following Sequence using DIT-FFT algorithm

$$x(n) = \begin{cases} 1 & \text{for } -3 \le n \le 3 \\ 0 & \text{Otherwise.} \end{cases}$$ [Page: 26] (8)

12. (a) Discuss the steps in the design of IIR filter using Bilinear transformation for any one type of filter. [Page: 49] (16)

Or

(b) Convert the following pole-zero IIR filter into a lattice ladder structure. (16)

$$H(z) = \frac{1 + 2z^{-1} + 2z^{-2} + z^{-3}}{1 + \left(\frac{13}{24}\right)z^{-1} + \left(\frac{5}{8}\right)z^{-2} + \left(\frac{1}{3}\right)z^{-3}}$$

13. (a) (i) Explain briefly how the zeros in FIR filter is located. (7)

(ii) Using a rectangular window technique, design a low pass filter which pass band gain of unity, cut off frequency of 1000Hz and working at a sampling frequency of 5 KHz. The length of the impulse response should be 7. [Page: 78] (9)

Or

(b) Consider an FIR lattice filter with co-efficients $K_1 = 1/2$; $K_2 = 1/3$; $K_3 = 1/4$

Determine the FIR filter Coefficients for the direct form Structure. (16).

14. (a) (i) Discuss the Various Common methods of quantization (8)

(ii) Explain the finite word length effects in FIR digital filters. (8)

Or

(b) Describe the quantization in floating point realization of IIR digital filters. (16)

15. (a) (i) Explain the implementation steps in Speech Coding Using transform Coding (8)

(ii) Discuss the design steps involved in the implementation of multistage Samplingrate Converter. [Page: 124] (8)

Or

(b) Explain the efficient implementation of polyphase decimator and interpolator [Page: 133] (16)

B.E/ B.Tech. DEGREE EXAMINATION, MAY/JUNE 2013

Fifth Semester

Electronics And Communication Engineering.

EC 2302 - DIGITAL SIGNAL PROCESSING.

(Regulation - 2008).

Time: Three hours Maximum: 100 marks

PART A - (10 x 2 = 20 marks)

1. Find the 4-point DFT of the sequence $x(n) = \{1, 1, -1, -1\}$. [Page: 16]

2. What is meant by in place Computation? [Page: 12]

3. Mention the advantages of Cascade realization.

4. Convert the given analog transfer function $H(s) = \frac{1}{s+a}$ into digital by impulse invariant method. [Page: 46]

5. List out the advantages and disadvantages of FIR filters. [Page: 71]

6. Write the equation of Hamming Window function. [Page: 72]

7. What are the two types of quantization employed in digital system? [Page: 94]

8. Define Zero input limit Cycle Oscillations. [Page: 96]

9. What is Anti imaging filter? [Page: 117]

10. Give the Applications multirate DSP. [Page: 119]

PART B - (5 × 16 = 80 marks)

11. (a) (i) State the following properties of DFT.

(1) Time Reversal.

(2) Parseval's Theorem. (8)

(ii) Perform the linear Convolution of the given sequences $x(n) = \{1, -1, 1, -1\}$, $h(n) = \{1, 2, 3, 4\}$ using DFT method (8) [Page: 19]

Or

(b) Derive the butterfly diagram of 8point radix-2 DIF-FFT algorithm and fully label it.

12. (a) A desired low pass filter with the following specification is,

$$0.8 \leq |H(\omega)| \leq 1.0; \quad 0 \leq \omega \leq 0.2\pi.$$

$$|H(\omega)| \leq 0.2; \quad 0.3\pi \leq \omega \leq \pi.$$

Design Butterworth digital filter using impulse invariant transformation. [Page: 49]

Or

(b) (i) Obtain the Cascade form realization of the digital System. [Page: 40 (8)

$$y(n) = \frac{3}{4}y(n-1) - \left(\frac{1}{8}\right)y(n-2) + \frac{1}{3}x(n-1) + x(n)$$

(ii) Convert the given Analog filter with a transfer function $H(s) = \frac{2}{(s+1)(s+2)}$ into a digital IIR filter using bilinear transformation Assume $T = 1$ Sec. [Page: 48] (8)

13. (a) (i) Determine the frequency response of FIR filter defined by $y(n) = 0.25x(n) + x(n-1) + 0.25x(n-2)$. Calculate the phase delay and group delay (8)

(ii) Discuss the design procedure of FIR filter using frequency Sampling method. (8) [Page: 90]

Or

(b) Design a FIR filter with the following desired Specification

$$H_d(e^{j\omega}) = \begin{cases} 0; & -\frac{\pi}{4} \le \omega \le \frac{\pi}{4} \\ e^{-j2\omega}; & \frac{\pi}{4} \le |\omega| \le \pi \end{cases}$$

using a Hanning Window with $N = 5$. (16) [Page: 80]

14. (a) (i) Represent the following numbers in floating point format with five bits for mantissa and three bits for exponent.

(1) 7_{10} (2) 0.25_{10} (3) -7_{10} (4) -0.25_{10} [Page:100] (8)

(ii) Draw the product quantization noise model of Second Order IIR System (8)

Or

(b) (i) Explain how Signal Scaling is used to prevent Overflow limit Cycle in the digital filter implementation with an [Page:114] (8)

Example.

(ii) Determine the dead band of the System $y(n) = 0.2\,y(n-1) + 0.5\,y(n-2) + x(n)$ Assume 8 bits are used for Signal representation [Page:107] (8)

15. (a) (i) Explain the multistage implementation of Sampling rate Conversion with block diagram. [Page:124] (8)

(ii) A Signal $x(n)$ is given by $x(n) = \{0, 1, 2, 3, 4, 5, 6, 0, 1, 2, 3, \ldots\}$ (8)

(1) Obtain the decimated Signal with a factor of 2.

(2) Obtain the interpolated signal with a factor of 2.

Or

(b) Explain sampling rate increase by an interger factor I and derive the input - Output relationship in both time and frequency domains. [Page: 125] (16)

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2012.

Fifth Semester.

Electronics and Communication Engineering.

EC 2302 - DIGITAL SIGNAL PROCESSING.

(Regulation - 2008)

Time: Three hours Maximum: 100 marks

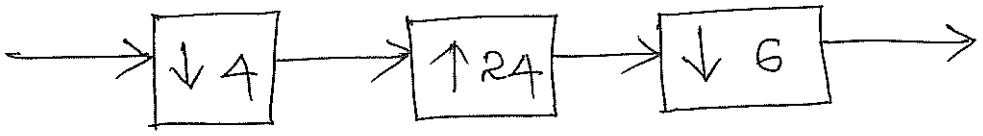
PART - A (10 × 2 = 20 marks)

1. What is twiddle factor? [Page:13]
2. How many stages of decimations are required in the case of a 64 point radix 2 DIT FFT algorithm? [Page:10]
3. Why is the butterworth response called a maximally flat response? [Page:37]
4. What is frequency warping? [Page:38]
5. What are the features of FIR filter design using the Kaiser approach?
6. Draw the direct form implementation of the FIR system having difference equation $y(n) = x(n) - 2x(n-1) + 3x(n-2) - 10x(n-6)$.
7. What are limit cycle oscillations?

8. What is dead band of a filter? [Page: 95]

9. What is decimation? [Page: 116]

10. Find the expression for the following multirate system.



PART B – (5 X 16 = 80 marks)

11. (a) (i) Differentiate DFT from DTFT (4) [Page: 1]

(ii) Compute an 8 point DFT of the sequence $x(n) = \{1, 0, \underset{\uparrow}{1}, -1, 1, -1, 0, 1\}$ [Page: 18] (12)

Or

(b) (i) Prove that FFT algorithms help in reducing the number of computations involved in DFT computation. [Page: 10] (6)

(ii) Compute a 8 point DFT of the sequence using DIT-FFT algorithm.

$x(n) = \{1, 2, \underset{\uparrow}{3}, 2, 1, 0\}$ [Page: 26] (10)

12. (a) (i) Explain the procedure for designing Analog filters using the Chebysher Approximation. [Page: 455] (6)

(ii) Convert the following Analog transfer function into digital using impulse invariant mapping with $T = 1$ Sec.

$$H(s) = \frac{3}{(s+3)(s+5)}$$ [Page: 46] (10)

Or

(b) (i) Design a digital Second Order low pass butterworth filter with Cut off frequency 2200 Hz using bilinear transformation Sampling rate is 8000 Hz. [Page: 49] (8)

(ii) Determine the Cascade form and parallel form implementation of the System governed by the transfer function. [Page: 40]

$$H(z) = \frac{(1+z^{-1})(1-5z^{-1}-z^{-2})}{(1+2z^{-1}+z^{-2})(1+z^{-1}+z^{-2})}$$ (8)

13. (a) Design an FIR low pass digital filter by using the frequency Sampling method for the following Specifications

Cut off frequency = 1500 Hz.

Sampling frequency = 15000 Hz.

Order of the filter $N = 10$. [Page: 90]

Filter length required $L = N + 1 = 11$.

Or

(b) (i) Explain with neat sketches the implementation of FIR filters in the

(1) Direct form.

(2) Lattice form. (6)

(ii) Design a digital FIR band pass filter with lower Cut off frequency 2000 Hz and Upper Cut off frequency 3200 Hz using Hamming window of length $N = 7$. Sampling rate is 10000 Hz. [Page: 84] (10)

14. (a) (i) What is quantization of Analog signals? Derive the Expression for the quantization error. (8)

(ii) Explain Coefficient quantization in IIR filter. [Page: 113] (8)

Or.

(b) (i) How to prevent limit Cycle Oscillations? Explain. (8)

(ii) What is meant by Signal Scaling? Explain. [Page: 114] (8)

15. (a) (i) Explain Sampling rate Conversion by a rational factor and derive input and Output relation in both time and frequency domain. [Page: 125] (10)

(ii) Explain the multistage implementation of Sampling rate Conversion. [Page: 124] (6)

Or

(b) (i) Explain the design of a narrow band filter using Sampling rate Conversion. (8)

(ii) Explain the Application of Sampling rate Conversion in Sub-band Coding. (8)

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2012.

Fifth Semester

Electronics and Communication Engineering

EC 2302 – DIGITAL SIGNAL PROCESSING.

Time: Three hours Maximum: 100 marks

PART A – (10×2 = 20 marks)

1. Distinguish between discrete time fourier transform and Discrete fourier transform. [Page: 11]

2. Determine the number of multiplications required in the computation of 8-point DFT using FFT. [Page: 10]

3. Compare Butterworth with Chebyshev filters. [Page: 36]

4. What is known as prewarping in digital filters. [Page: 36]

5. Define Gibbs phenomenon. [Page: 72]

6. What are Called Symmetric and Anti-symmetric FIR filters? [Page: 71]

7. Compare truncation with rounding errors. [Page: 95]

8. What is known as Overflow Oscillations? [Page: 97]

9. Give Applications of multirate Sampling. [Page: 119]

10. What are Called polyphase filters? [Page: 117]

PART B – (5 × 16 = 80 marks)

11. (a) Explain the following properties of DFT. (16)

(i) Linearity (ii) Complex Conjugate property.

(iii) Circular Convolution (iv) Time Reversal.

Or

(b) (i) Explain Decimation in time FFT (DITFFT) algorithm for 8 point DFT Computation. [Page: 21] (10)

(ii) Find the DFT of $x(n) = \{1, 1, 0, 0\}$ (6) [Page: 16]

12. (a) Design a Butterworth filter using the bilinear transformation for the given Specification. [Page: 49] (16)

$$0.8 \leq |H(e^{j\omega})| \leq 1 \quad ; \quad 0 \leq \omega \leq 0.2\pi$$

$$|H(e^{j\omega})| \leq 0.2 \quad ; \quad 0.6\pi \leq \omega \leq \pi$$

Or

(b) (i) Find $h(n)$ using impulse invariant method for the given $H(s) = \frac{10}{s^2 + 7s + 10}$. Assume T = 1 sec (8) [Page: 46]

(ii) Obtain Direct form I and Direct form II Realization of $H(z) = \frac{1+2z^{-1}}{1-1.5z^{-1}+0.4z^{-2}}$ (8)

[Page: 40]

13. (a) (i) Explain the frequency Sampling method of FIR filter design (10)

(ii) Determine the Direct form realization of System function.

$$H(z) = 1+2z^{-1}-3z^{-2}-4z^{-3}+5z^{-4} \quad (6)$$

Or

(b) Design a FIR low pass filter having the following Specifications using Hanning Window.

$$H_a(e^{jw}) = \begin{cases} 1 & \text{for } -\frac{\pi}{6} \le |w| \le \frac{\pi}{6} \\ 0 & \text{Otherwise} \end{cases} \quad (16)$$

And $N = 7$.

14. (a) Explain the following:

(i) Coefficient Quantization Error.

(ii) Product Quantization Error.

(iii) Signal Scaling.

(iv) Truncation And Rounding. (16)

Or

(b) (i) Compare floating point with fixed point Arithmetic. (4)

(ii) What is called Quantization noise? Derive the Expression for quantization noise power. [Page: 101] (12)

15. (a) Explain the Concept of decimation and interpolation of discrete time signals. [Page: 120] (16)

Or

(b) (i) Explain in detail the multistage implementation of Sampling rate Conversion. [Page: 124] (8)

(ii) Explain Subband Coding in detail. (8)

B.E. / B. Tech. DEGREE EXAMINATION

NOVEMBER / DECEMBER 2011.

Fifth Semester.

Electronics And Communication Engineering.

EC 2302 - DIGITAL SIGNAL PROCESSING.

(Regulation 2008)

Time: Three hours Maximum: 100 marks.

PART A - (10 X 2 = 20 marks)

1. Distinguish between DFT and DTFT. [Page: 11]
2. What is Zero padding? What are its uses? [Page: 10]
3. List the properties of Chebyshev filter. [Page: 36]
4. Draw the direct form structure of IIR filter. [Page: 40]
5. What are the desirable Characteristics of Window? [Page: 70]
6. What are the properties of IIR filter? [Page: 71]
7. What is Overflow Oscillations? [Page: 97]
8. What are Advantages of floating point Arithmetic?
9. What is multirate Signal processing? [Page: 116]
10. What is meant by down Sampling and Upsampling? [Page: 116]

PART B (5 × 16 = 80 marks)

11. (a) (i) Determine the N-point DFT of the following sequences (6)

(1) $x(n) = \delta(n)$ (2) $x(n) = \delta(n-n_0)$

(ii) Compute 8-point DFT of the sequence $x(n) = \{0, 1, 2, 3, 4, 5, 6, 7\}$ using radix-2 DIF algorithm. (10)

[Page: 26]

Or

(b) Compute the linear convolution of the finite duration sequences $h(n) = \{1, 2\}$ and $x(n) = \{1, 2, -1, 2, 3, -2, -3, -1, 1, 1, 2, -1\}$ by overlap add method. (16)

[Page: 30]

12. (a) Design a digital butterworth filter using impulse invariance method satisfying the constraints. Assume $T = 1$ Sec. [Page: 49]

$$0.8 \leq |H(e^{j\omega})| \leq 1 \; ; \; 0 \leq \omega \leq 0.2\pi.$$

$$|H(e^{j\omega})| \leq 0.2 \; ; \; 0.6\pi \leq \omega \leq \pi.$$

Or

(b) Obtain the direct form I, direct form

II And Cascade form realization of the following System functions. [Page: 40]

$$y(n) = 0.1\,y(n-1) + 0.2\,y(n-2) + 3\,x(n) + 3.6\,x(n-1) + 0.6\,x(n-2).$$

13. (a) Design an ideal differentiator with frequency response,

$H(e^{jw}) = jw \; ; \; -\pi \le w \le \pi$ Using Hamming Window with $N = 8$. (16)

Or

(b) Design an ideal high pass filter using hanning window with a frequency response. Assume $N = 11$. [Page: 80] (16)

$$H_d(e^{jw}) = 1 \; ; \; \frac{\pi}{4} \le |w| \le \pi$$

$$= 0 \; ; \; |w| \le \pi/4$$

14. (a) With respect to finite word length effects in digital filters, with Examples discuss about

(i) Overflow limit Cycle Oscillation (10)

(ii) Signal Scaling [Page: 114] (6)

Or

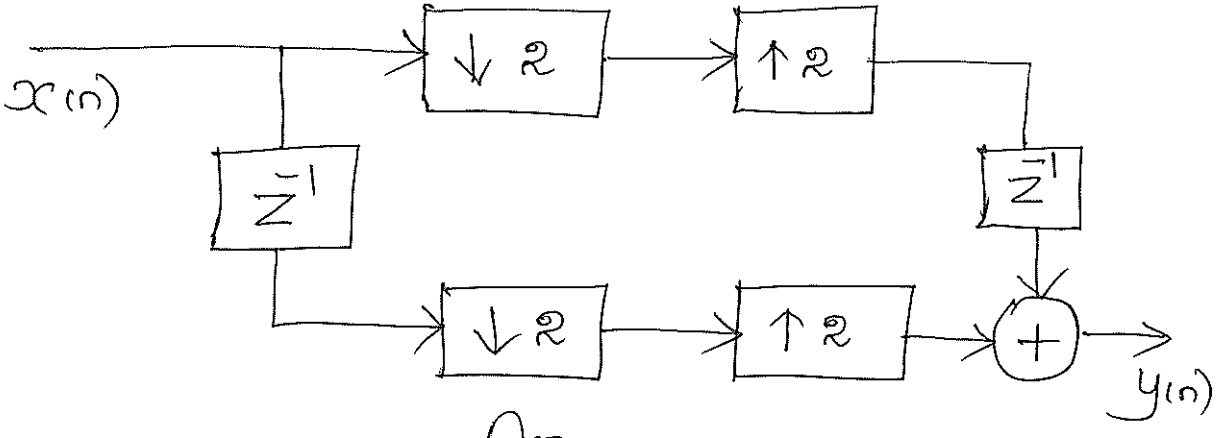
(b) (i) Distinguish between fixed point And floating point Arithmetic. (6)

(ii) Consider a Second Order IIR filter with

$$H(z) = \frac{1.0}{(1-0.5z^{-1})(1-0.45z^{-1})}$$

Find the effect On Quantization On pole location of the given System function in direct form and in Cascade form. Assume $b = 3$ bits. [Page:113] (10)

15. (a) For the multirate System. Shown in figure. Find the relation between $x(n)$ and $y(n)$. (16)



Or

(b) Explain the efficient transversal Structure for decimator and interpolator (16)

B.E. / B. Tech. DEGREE EXAMINATION APRIL / MAY 2011.

FIFTH SEMESTER.

Electronics and Communication Engineering

EC 2302 - DIGITAL SIGNAL PROCESSING.

Time: Three hours Maximum: 100 marks

PART A (10 × 2 = 20 marks)

1. State the Advantages of FFT over DFTs [Page:10]
2. What is meant by bit reversal? [Page:11]
3. Why do we go for Analog Approximation to design a digital filter?
4. Give Any two properties of Chebyshev filter. [Page:36]
5. State the properties of FIR filters. [Page:71]
6. What is meant by Gibb's phenomenon? [Page:72]
7. What is meant by fixed point Arithmetic? [Page:98] Give Example.
8. Explain the meaning of limit Cycle [Page:96] Oscillaton.
9. State the Various Applications of DSP. [Page:118]
10. What is Echo Cancellation?

PART- B (5 × 16 = 80 marks)

11. (a) With appropriate diagrams describe

(i) Overlap Save method. (8)

(ii) Overlap add method. (8)

[Page:30]

Or

(b) Explain radix 2 DIF FFT algorithm. Compare it with DIT- FFT algorithms. (16)

12. (a) Explain in detail butterworth filter approximation. [Page: 49] (16)

Or

(b) Explain the bilinear transform method of IIR filter design. What is warping effect? Explain the poles and zeros mapping procedure clearly. (16)

13. (a) Realize the system function $H(z) = \left(\frac{2}{3}\right) z + 1 + \left(\frac{2}{3}\right) z^{-1}$ by linear phase FIR structure. (16)

Or.

(b) Explain the designing of FIR filters [Page 74, 78, 90] (16)

14. (a) Explain the quantization process and errors introduced due to quantization. (16)

Or

(b) (i) Explain how reduction of product round off error is achieved in digital filters. [Page: 109] (8)

(ii) Explain the effects of Coefficient Quantization in FIR filters. [Page: 113] (8)

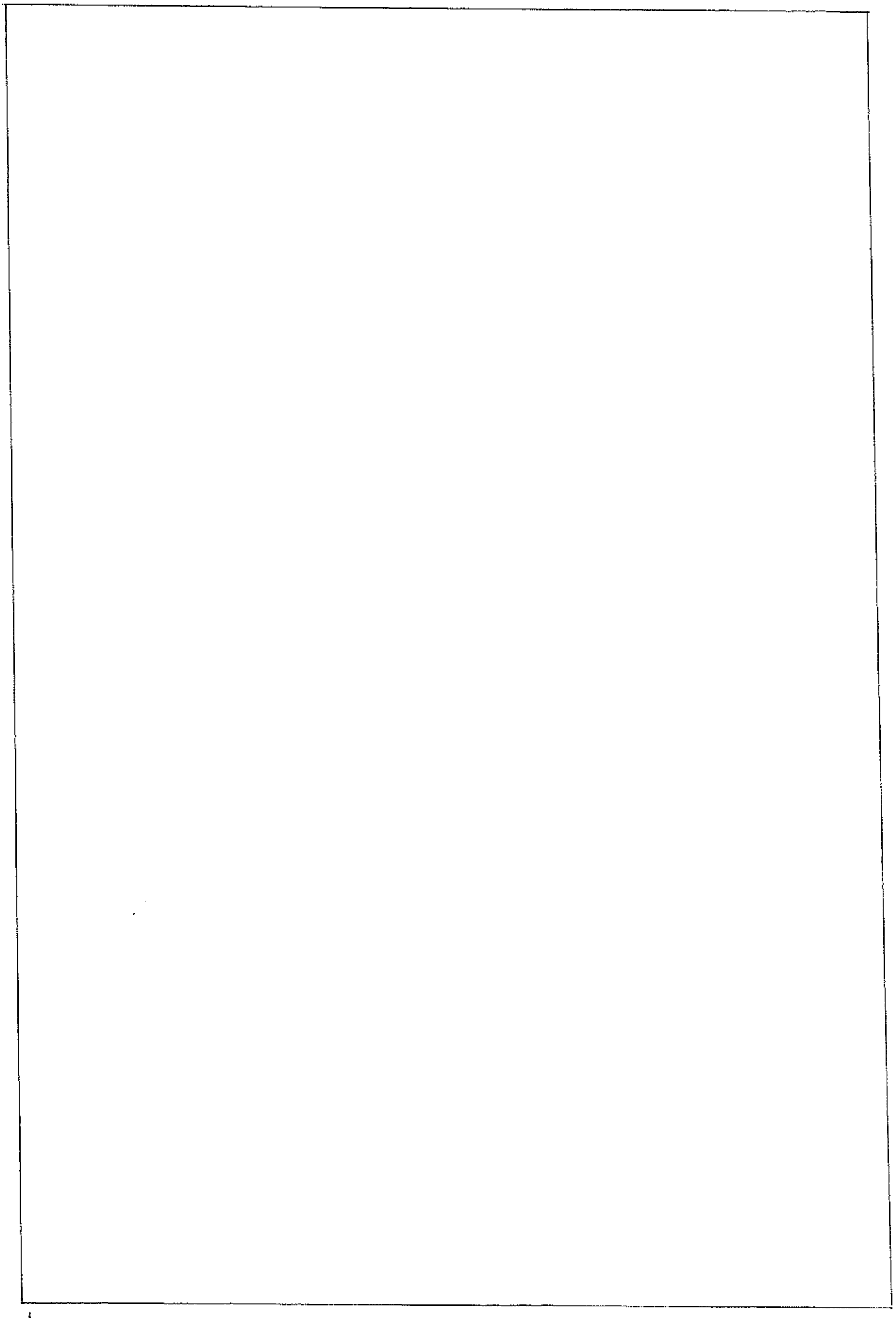
15. (a) (i) Explain how various Sound effects can be generated with the help of DSP. (10)

(ii) State the Applications of multirate Signal processing. [Page: 119] (6)

Or

(b) (i) Explain how DSP can be used for Speech processing. (8)

(ii) Explain in detail about decimation and interpolation. [Page: 120] (8)



B.E. / B. Tech. DEGREE EXAMINATION NOVEMBER / DECEMBER 2010.

Fifth Semester.

Electronics and Communication Engineering

EC 2302 – DIGITAL SIGNAL PROCESSING.

(Regulation 2008)

Time: Three hours Maximum: 100 marks.

PART A (10 × 2 = 20 marks)

1. Obtain the Circular Convolution of the following sequences $x(n) = \{1, 2, 1\}$: $h(n) = \{1, -2, 2\}$.

2. How many multiplications and additions are required to Compute N-point DFT using radix-2 FFT. [Page:10]

3. What is prewarping? [Page: 36]

4. What is the advantage of direct form II realization when Compared to direct form I realization? [Page: 39]

5. Give the equations for Hamming window and Black man window.

6. Determine the transversal structure of the system function $H(z) = 1 + 2z^{-1} - 3z^{-2} - 4z^{-3}$.

7. What is truncation? [Page: 95]

8. What is product quantization Error? [Page: 94]

9. What is decimation? [Page: 116]

10. What is Sub band Coding?

PART B (5 × 16 = 80 marks)

11. (a) (i) Compute the eight point DFT of the sequence

$$x(n) = \left\{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 0, 0, 0, 0\right\}$$

Using the radix-2 Decimation in time algorithm. [Page: 26] (10)

(ii) Explain Overlap add method for linear FIR filtering of a long sequence [Page: 30] (6)

Or

(b) (i) Compute the eight point DFT of the sequence.

$$x(n) = \begin{cases} 1; & 0 \le n \le 7 \\ 0; & \text{Otherwise} \end{cases}$$

By using the decimation in frequency FFT algorithm. [Page: 26] (10)

(ii) Summarize the properties of DFT (6)

12. (a) Determine the System function $H(z)$ of the Chebyshev's low pass digital filter with the specifications. (16)

α_p = 1 dB ripple in the pass band

$0 \le \omega \le 0.2\pi$.

α_s = 15 dB ripple in the stop band

$0.3\pi \le \omega \le \pi$.

Using bilinear transformation (assume $T = 1$ sec) [Page: 55]

Or

(b) Obtain the direct form I, direct form II, Cascade and parallel form realization for the System. [Page: 40] (16)

$$y(n) = -0.1\,y(n-1) + 0.2\,y(n-2) + 3x(n) + 3.6x(n-1) + 0.6x(n-2)$$

13. (a) Design an ideal high pass filter with a frequency response. [Page: 84]

$$H_d(e^{j\omega}) = \begin{cases} 1 & \text{for } \frac{\pi}{4} \le |\omega| \le \pi \\ 0 & \text{for } |\omega| \le \pi/4 \end{cases}$$

Find the values of $h(n)$ for $N = 11$ using hamming Window. Find $H(z)$ and determine

the magnitude response. (16)

Or

(b) (i) Determine the Coefficients $h(n)$ of a linear phase FIR filter of length $M=15$ which has a Symmetric Unit Sample Response and a frequency response that satisfies the Condition. [Page: 90] (10)

$$H_r\left(\frac{2\pi k}{15}\right) = \begin{cases} 1, & \text{for } k=0,1,2,3 \\ 0, & \text{for } k=4,5,6,7 \end{cases}$$

(ii) Obtain the linear phase realization of the System function.

$$H(z) = \frac{1}{2} + \frac{1}{3}z^{-1} + z^{-2} + \frac{1}{4}z^{-3} + z^{-4} + \frac{1}{3}z^{-5} + \frac{1}{2}z^{-6} \quad (6)$$

14. (a) Discuss in detail the errors resulting from rounding and truncation (16)

Or

(b) Explain the limit Cycle Oscillation due to product round off and Overflow errors. [Page: 107] (16)

15. (a) Explain the polyphase structure of decimator and interpolator (16)

Or [Page: 133]

(b) Discuss the procedure to implement digital filter bank using multirate Signal processing. (16)

